# Evolutionary coordination system for fixed-wing communications unmanned aerial vehicles: supplementary online material 

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#### Abstract

This document provides supplementary online material for the paper titled "Evolutionary coordination system for fixed-wing communications unmanned aerial vehicles" recently submitted to TAROS 2014. It mainly focuses on critical system components, namely the aerial vehicles kinematics model, link budget management and also addresses the issue of time synchronisation and algorithmic flow.


## 1 System components

In this section, two key components are described; a) the kinematics model that dictates the manoeuvrability of the aerial vehicles followed by b) the mathematical model that is used to measure the power consumption for the provided communication network.

### 1.1 Kinematics model

An aerial vehicle is treated as a point object in three-dimensional space with an associated direction vector. At each time step, the position of an aerial vehicle is defined by the latitude, longitude, altitude and heading ( $\phi_{c}, \lambda_{c}, h_{c}, \theta_{c}$ ) in a geographic coordination system. Preferably, a fixed-wing aerial vehicle flies according to a 6 DOF model of several restrictions, ranging from weight and drag forces to atmospheric phenomena, that affect its motion. However, as this work focuses on the coordination of the group of aerial vehicles with respect to the communication network, a simplified decoupled kinematics model based on simple turns is considered for the restrictions of both horizontal and vertical motions.


Fig. 1. (a) Horizontal motion; (b) Vertical motion.

Horizontal and vertical motions: For the horizontal flight of an aerial vehicle equal opposing lift $L$ and weight $W$ forces are required (shown in figure 1 b ). A centripetal force $F$ is subsequently required such that it will move at the horizontal and/or vertical planes. The wing's load factor of such a model is defined as the ratio $n=\frac{L}{W}$, with $n=1$ for horizontal (level) and $n>1$ for vertical (pulling up) flights, respectively. For a level flight with a horizontal turn, the wing's load factor is calculated by considering the following equations (dynamics depicted in figure 1 b , right):

$$
\begin{gather*}
W=L \cos \beta  \tag{1}\\
L \sin \beta=m \frac{u^{2}}{r}  \tag{2}\\
n=\frac{L}{W}=\frac{L}{L \cos \beta}=\frac{1}{\cos \beta}=\sec \beta \tag{3}
\end{gather*}
$$

with $\beta$ being the desired bank angle, $r$ the turn radius of the horizontal turn, $m$ the mass and $g$ the acceleration gravity (with $W=m g$ ). The centripetal force $F$ is now written as

$$
\begin{equation*}
F=\sqrt{L^{2}-W^{2}}=\sqrt{(n W)^{2}-W^{2}}=W \sqrt{n^{2}-1} \tag{4}
\end{equation*}
$$

notice that as $W$ is mass $m$ times the acceleration gravity $g$, the previous equation can be expanded to

$$
\begin{equation*}
F=m g \sqrt{n^{2}-1} \tag{5}
\end{equation*}
$$

combining equations 1 and 2, the bank angle is written as

$$
\begin{equation*}
\tan \beta=\frac{1}{g} * \frac{u^{2}}{r} \tag{6}
\end{equation*}
$$

from that, the turn radius $r$ is initially expressed as

$$
\begin{equation*}
r=\frac{u^{2}}{g} \frac{1}{\tan \beta} \tag{7}
\end{equation*}
$$

The trigonometric equivalence $\sec \beta=1+\tan ^{2} \beta$ allows the following expansion to the turn radius equation

$$
\begin{equation*}
\tan \beta=\sqrt{\sec \beta-1}=\sqrt{\left(\frac{1}{\cos \beta}\right)^{2}-1}=\sqrt{n^{2}-1} \tag{8}
\end{equation*}
$$

now, equation 7 is further expanded to

$$
\begin{equation*}
r=\frac{1}{g} \frac{u^{2}}{\sqrt{n^{2}-1}} \tag{9}
\end{equation*}
$$

Relating the rate of turn in a horizontal plane to the velocity $u$ and turn radius $r$, and combined with equation 9 , the following rate of turn is formed

$$
\begin{equation*}
\dot{\theta}=\frac{u}{r}=\frac{g \sqrt{n^{2}-1}}{u} \tag{10}
\end{equation*}
$$

thus,

$$
\begin{equation*}
\dot{\theta}=\frac{\Delta \theta}{\Delta t}=\frac{g \sqrt{n^{2}-1}}{u} \tag{11}
\end{equation*}
$$

A higher bank angle generates a higher wing's load factor, which ultimately creates a higher centripetal force that allows a tight turn to be performed.

Furthermore, the new heading of the aerial vehicle after a time step $\Delta t$ of a horizontal manoeuvre is written as

$$
\begin{equation*}
\theta_{(t+\Delta t)}=\theta_{t}+\Delta \theta=\theta_{t}+(\dot{\theta} \Delta t) \tag{12}
\end{equation*}
$$

and the distance travelled within $\Delta t$ is calculated by

$$
\begin{equation*}
\Delta d=v * \Delta t \tag{13}
\end{equation*}
$$

The new latitude $\phi_{n}$ and longitude $\lambda_{n}$ in $\Delta t$ can be estimated using spherical trigonometry as follows

$$
\begin{gather*}
\phi_{(t+\Delta t)}=\arcsin \left(\sin \left(\phi_{t}\right) \cos \left(\frac{\Delta d}{R}\right)+\cos \left(\phi_{t}\right) \sin \left(\frac{\Delta d}{R}\right) \cos \left(\theta_{t}\right)\right)  \tag{14}\\
\lambda_{(t+\Delta t)}=\lambda_{t}+\arctan \left(\sin \left(\theta_{t}\right) \sin \left(\frac{\Delta d}{R}\right) \cos \left(\phi_{t}\right)\right.  \tag{15}\\
\left.\cos \left(\frac{\Delta d}{R}\right)-\sin \left(\phi_{t} \sin \phi_{(t+\Delta t)}\right)\right)
\end{gather*}
$$

where $R$ is Earth's radius.

Similar methods are adopted for the vertical motion. The rate of climb and descent angle $\chi$, as shown in figure 1a, is expressed by

$$
\begin{equation*}
\dot{\chi}=\frac{\Delta \chi}{\Delta t}=\frac{g(n-1)}{u} \tag{16}
\end{equation*}
$$

For a time step $\Delta t$ the new climb or descent angle of the aerial vehicleat time $t+\Delta t, \chi_{(t+\Delta t)}$ is estimated by adding to the current $\chi_{t}$, as shown by

$$
\begin{equation*}
\chi_{(t+\Delta t)}=\chi_{t}+\Delta \chi \tag{17}
\end{equation*}
$$

and is constrained to the maximum climb and descent angle of the aerial vehicle based on its mechanical characteristics. Therefore, at every $\Delta t$ the altitude change is expressed by

$$
\begin{equation*}
\Delta d=(u \Delta t) \times \tan \left(\chi_{(t+\Delta t)}\right) \tag{18}
\end{equation*}
$$

For security as well as reasons regarding practical constraints such as altitude ceilings, airspace limitations or terrain avoidance criteria, the kinematics model is designed to allow flying within a pre-defined flying corridor. Therefore, the model denies altitude additions or subtractions in case the maximum or minimum permitted altitude is reached.

Manoeuvres: An aerial vehicle may either perform a turn circle manoeuvre with a tight bank angle in order to keep its current position, or implement a manoeuvre solution generated by the EA. Taking inspiration from the Dubins curves and paths [1], when implemented a manoeuvre solution will generate a trajectory consisting of three segments, as depicted in figure 2. Each segment can be a straight line, turn left or turn right curve, depending on the given bank angle.


Fig. 2. One manoeuvre of three segments of different duration and bank angles, between the starting point A and finishing point B. Direction of flying is dictated by the bank angle.

The EA is free to select the duration for any of the segments as long as the overall remains equal to one revolution time of a turn circle manoeuvre. This strategy ensures synchronisation between the aerial vehicles. With a tighter bank angle of 75 degrees and a constant speed of 110 knots, one revolution time is approximately 6 seconds. The aerial vehicles perform 2 turn circle manoeuvres before they are allowed to implement the latest generated solution from the

EA. Obviously time is an important aspect of the system as the EA runs online. This time window ensures that the artificial evolution will have reached a satisfactory result, while at the same time the aerial vehicles will fly in a controlled and synchronized way, keeping their previous formation. Furthermore, the time window sure that the aerial vehicles will have enough time to exchange fresh GPS data and ultimately communicate the resulting solution on time.

### 1.2 Link budget

Communication is achieved by maintaining communication links between the aerial backbone and as many ground-based vehicles as possible. The communication links are treated independently and a transmission is considered successful when the transmitter is able to feed its antenna with enough power, such that it satisfies the desirable quality requirements. It is assumed that aerial vehicles are equipped with two radio antennae. One isotropic able to transmit to all directions and a horn-shaped one able to directionally cover an area on the ground. It is also assumed that all vehicles are equipped by a Global Positioning System (GPS) and can broadcast information about their current position and directionality at a reasonable interval (default 3 seconds). In this section, focus is primarily given to the communication between aerial vehicles and ground-based vehicles using the former horn-shaped antennae, as it dictates the effectiveness of the communication coverage of the mission and the power consumption of a flying mission.

No matter what the modulation and demodulation scheme is applied at the higher protocol levels, a link is considered of a good quality if the ratio of the energy per bit of information $E_{b}$ to the thermal noise in 1 Hz bandwidth $N_{0}$ (normalized signal to noise ratio ${ }^{E_{b}} / N_{0}$ ) is maintained. The transmitting power $P_{t}$ that an aerial vehicle is required to feed to its horn-shaped antenna, in order to cover a ground-based vehicle in distance $d$ is expressed by the following version of Friis equation

$$
\begin{equation*}
P_{t}=p \times d^{2} R_{b} \frac{E_{b}}{N_{0}} \frac{1}{G_{r} G_{t}}\left(\frac{4 \pi f}{c}\right)^{2} T_{s y s} K \tag{19}
\end{equation*}
$$

- $R_{b}$ is the desired data rate on the link (bit/s),
- $E_{b} / N_{0}$ target ratio of energy in one bit to the noise in 1 Hz ,
- $G_{r}$ receiver's antenna gain (assuming omnidirectional),
$-G_{t}$ transmitter's antenna gain equal to $\frac{2 \eta}{1-\cos (H P B W / 2)}$, where $\eta$ and $H P B W$ are the efficiency and the half-power beamwidth angle of the horn-shape antenna,
- $T_{\text {sys }}$ the total system noise temperature,
- $K$ is the Boltzmann K constant,
$-p$ is coverage profile (will be explained below), and
$-d$ is the slant range defining the distance between the aerial vehicle and the ground-based vehicle on the ground

Here, most of the terms are known and remain constant during the mission. In order for a ground-based vehicle to be covered, it needs to lie within the footprint of at least one aerial vehicle.


Fig. 3. Slant range $d$ and angle $h$ of a communication link.

As shown in figure 3, a footprint is determined by the altitude of the aerial vehicle as well as its antenna's half-power beamwidth angle. The higher the aerial vehicle flies, the wider its footprint is on the ground, the greater the area covered. The slant angle $h$ of the ground-based vehicle with respect to the aerial vehicle is calculated by applying spherical trigonometry (see below) on the available GPS data that each network participant broadcasts. The following piecewise function is then used to decide weather a ground-based vehicle lies within the footprint.

$$
L(p)=\left\{\begin{array}{l}
1: h<^{H P B W} / 2 \\
0: h \geq^{H P B W} / 2
\end{array}\right.
$$

The longer the distance between the transmitter and the receiver, the higher the signal power required to support the communication. In terms of equations, lets consider the scenario where an aerial vehicle is transmitting to a ground-based vehicle.

Knowing $R$, Earth's radius, the distances of the transmitting antenna $T x$ and the receiving antenna $R x$ from the centre of the Earth are found by

$$
\begin{equation*}
T x=R+h_{t x}, \quad R x=R+h_{r x} \tag{20}
\end{equation*}
$$

where $h_{t x}$ and $h_{r x}$ are the altitudes of the aerial vehicle and ground-based vehicle respectively. The slant range $d$ for the equation 19 is therefore found as

$$
\begin{equation*}
d=\sqrt{T x^{2}+R x^{2}-2 \cos (a) T x R x} \tag{21}
\end{equation*}
$$

subsequently the slant angle $h$, as depicted in figure 3, required in order to estimate the profile $p$ value for the equation 19 is calculated as

$$
\begin{equation*}
h=\arcsin \left(\frac{T x}{d} \sin (a)\right) \tag{22}
\end{equation*}
$$

with $a$ being the central angle (figure 4), calculated by

$$
\begin{equation*}
a=\arccos \left(\sin \left(T x_{\phi}\right) \sin \left(R x_{\phi}\right)+\cos \left(T x_{\phi}\right) \cos \left(R x_{\phi}\right) \cos \left(R x_{\lambda}-T x_{\lambda}\right)\right) \tag{23}
\end{equation*}
$$



Fig. 4. Slant range $d$ and angle $h$ of a communication link wrt. the central angle $a$. Notice that the system allows non-zero altitudes for the ground-based vehicles.

## 2 Time synchronisation and algorithm flow

The concept of time and time synchronisation plays a important role in the system. As mentioned above, the aerial vehicles are allowed to perform two types of manoeuvres; i) a turn circle manoeuvre of a fixed bank angle, and ii) the resulting evolved manoeuvre generated by the EA decision unit.

As the system does not consider noise at the current, abstract stage (e.g., wind force and variations to cruising speed), an aerial vehicle is expected to perform perfect circles and always reach the same final latitude, longitude, and altitude when completing a revolution. In terms of a EA generated manoeuvres, all aerial vehicles are expected to complete their flying at an equal time, due to the Dubins manoeuvres' equal durations. Since the resulting decision that dictates the next move is communicated from the master aerial vehicle to the rest


Fig. 5. Process flow for a master aerial vehicle.
of the group using the network, it is understood that there are delays that may affect the transmissions, such that not all of the aerial vehicles will be informed on time. This brings a synchronisation problem which is addressed by increasing the number of revolutions required before a Dubins path is implemented by the aerial vehicles. The significance of this rule is two-fold. Not only it allows all aerial vehicles to successfully receive the next manoeuvre information, depending on the number of repeated revolutions set, it also ensures that the EA has enough time to reach a solution.

After solving the time synchronisation issues the EA is able to run in parallel with the rest of the controller, which is responsible for keeping the aerial vehicle flying in order to complete its previous manoeuvre, either a turn circle or a Dubins manoeuvre. Figure 5 depicts the algorithm flow at the master aerial vehicle. At every time step $d t$, the algorithm takes a step further to the previous unfinished manoeuvre as long as it is not completed. If no Dubins manoeuvre is available and a turn circle manoeuvre is already performed, the master aerial vehicle interacts with the EA decision unit in order to receive a freshly evolved solution (set of manoeuvres for all the group of aerial vehicles). Notice that in case the EA has not been successful in evolving a good solution, the master aerial vehicle communicates turn circle manoeuvres to the rest group. This allows synchronisation between the aerial vehicles as they are bound to make a revolution, whilst giving more time to the EA algorithm to produce a result.

Since a non-master aerial vehicle (shown in figure 6) does not interact with an EA decision unit at the current stage of the system, it does construct and


Fig. 6. Process flow for a non-master aerial vehicle.
perform turn circle manoeuvres only when no Dubins (or turn circle manoeuvre) is received by the master aerial vehicle. This feature is rather unusable as the aerial vehicles will always find a received manoeuvre due to the perfect network conditions discussed before. Nevertheless, in reality this will allow a non-master aerial vehicle to continue flying in a controlled attitude and give it time to synchronize with the rest of the group. It is empirically found that a double turn circle manoeuvre of 6 seconds ( 75 degrees bank angle) gives enough time to the EA decision unit to produce reasonable results for a group of 6 aerial vehicles. Ultimately, the group's flying pattern is as follows: turn circle $\rightarrow$ turn circle $\rightarrow$ Dubins $\rightarrow$ turn circle $\rightarrow$ turn circle $\rightarrow$ Dubins $\rightarrow$... etc.

## References

1. Lester E. Dubins. On plane curves with curvature. Pacific Journal of Mathematics, 11(2):471-481, 1961.
