Not for examination purposes

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Mathematical Formulae and Fundamental Constants

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Printed 21/05/2007

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Differentiation

$y = f(x)$	$\frac{dy}{dx} = f'(x)$
k, constant	0
x^n, any constant n	nx^{n-1}
e^x	e^x
$\ln x = \log_e x$	$\frac{1}{x}$
sinx	cosx
cosx	-sinx
tanx	$\sec^2 x$
cotx	$-\mathrm{cosec}^2 x$
sinhx	coshx
coshx	sinhx

Integration

$f(x)$	$\int f(x)dx = F(x) + c$
k, constant	kx + c
x^n, $(n \neq -1)$	$\frac{1}{n+1} x^{n+1}$
e^x	$e^x + c$
$\frac{1}{x}$	$\ln x + c \quad (x > 0)$ $\ln(-x) + c \quad (x < 0)$
$\log_e x$	$x\log_e x - x$
sinx	-cosx + c
cosx	sinx + c
tanx	ln(secx) + c
cotx	ln(sinx) + c
sinhx	coshx + c
coshx	sinhx + c
$\frac{1}{\sqrt{(1-x^2)}}$	$\sin^{-1}x + c$
$\frac{1}{1+x^2}$	$\tan^{-1}x + c$

Laws of Indices

$a^m a^n = a^{(m+n)}$
$\frac{a^m}{a^n} = a^{(m-n)}$
$(a^m)^n = a^{mn}$
$a^0 = 1$
$a^{-m} = \frac{1}{a^m}$
$a^{1/n} = \sqrt[n]{a}$

Complex variables

z = complex variable,
x, y = real variables
r = amplitude (real)
θ = phase (real)
|z| = modulus of z
arg z = argument of z
z* = complex conjugate of z
n = integer

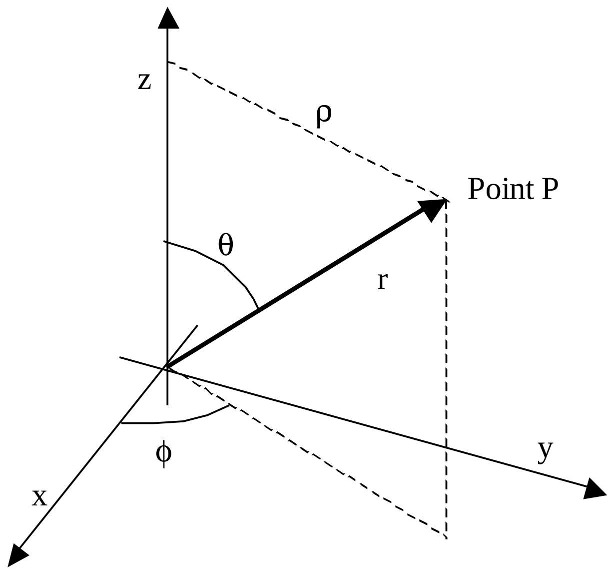
Cartesian form	$z = x + jy$
Polar form	$z = re^{j\theta} = r(\cos\theta + j\sin\theta)$
Modulus	$\|z\| = r = (x^2 + y^2)^{1/2}$
Argument	$\theta = \arg z = \arctan(y/x)$
Complex conjugate	$z^* = x - jy = re^{-j\theta}$
de Moivre’s Theorem	$(\cos\theta + j\sin\theta)^n = \cos n\theta + j\sin n\theta$

Geometry

For a circle of radius r and diameter d,
Circumference = $2\pi r = \pi d$
Area = $\pi r^2 = \pi d^2/4$

For a sphere of radius r,
Surface area = $4\pi r^2$
Volume = $\frac{4}{3}\pi r^3$
For a cylinder of radius r and height h
Surface area = $2\pi rh$
Volume = $\pi r^2 h$

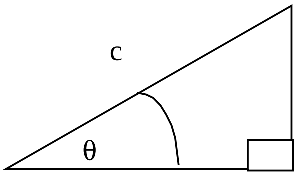
Common three-dimensional co-ordinate systems



Cartesian	**Cylindrical**	**Spherical**
x	$\rho\cos\phi$	$r\sin\theta\cos\phi$
y	$\rho\sin\phi$	$r\sin\theta\sin\phi$
z	Z	$r\cos\theta$

Trigonometry

$360^0 = 2\pi$ radians



$\sin\theta = b/c$ $\cos\theta = a/c$ $\tan\theta = b/a$

$\sin 45^0 = 1/\sqrt{2}$ $\cos 45^0 = 1/\sqrt{2}$ $\tan 45^0 = 1$

$\sin 30^0 = 1/2$ $\cos 30^0 = \sqrt{3}/2$ $\tan 30^0 = 1/\sqrt{3}$

$\sin 60^0 = \sqrt{3}/2$ $\cos 60^0 = 1/2$ $\tan 60^0 = \sqrt{3}$

$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$
$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$
$\sin^2 A + \cos^2 A = 1$
$\cos^2 A - \sin^2 A = \cos 2A$
$\sin 2A = 2\sin A\cos A$

$\cos(-A) = \cos A$
$\sin(-A) = -\sin A$
$\sin(\pi + A) = -\sin A$
$\sin(\pi - A) = \sin A$
$\sin(\pi/2 + A) = \cos A$
$\sin(\pi/2 - A) = \cos A$
$\cos(\pi + A) = -\cos A$
$\cos(\pi - A) = -\cos A$
$\cos(\pi/2 + A) = -\sin A$
$\cos(\pi/2 - A) = \sin A$

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Matrices and Determinants

The 2 x 2 matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ has determinant

$$|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

and has an inverse

$$A^{-1} = \frac{1}{ad - bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

providing $ad - bc \neq 0$

The 3 x 3 matrix $A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$ has determinant

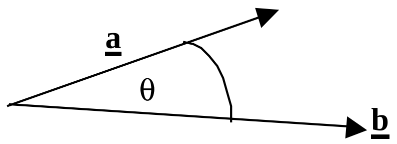
$$|A| = a_{11}(a_{22}a_{33} - a_{23}a_{32}) - a_{12}(a_{21}a_{33} - a_{23}a_{31}) + a_{13}(a_{21}a_{32} - a_{22}a_{31})$$

Vectors

If $\underline{\mathbf{r}}$ = x$\underline{\mathbf{i}}$ + y$\underline{\mathbf{j}}$ +z$\underline{\mathbf{k}}$ then |$\underline{\mathbf{r}}$| = $\sqrt{x^2 + y^2 + z^2}$

$\underline{\mathbf{a}}.\underline{\mathbf{b}} = |\underline{\mathbf{a}}||\underline{\mathbf{b}}|\cos\theta$

$\underline{\mathbf{a}} \times \underline{\mathbf{b}} = |\underline{\mathbf{a}}||\underline{\mathbf{b}}|\sin\theta\ \underline{\hat{\mathbf{e}}}$ where $\underline{\hat{\mathbf{e}}}$ is a unit vector perpendicular to the plane containing $\underline{\mathbf{a}}$ and $\underline{\mathbf{b}}$



For a scalar field f, $\nabla f = \frac{\partial f}{\partial x}\underline{\mathbf{i}} + \frac{\partial f}{\partial y}\underline{\mathbf{j}} + \frac{\partial f}{\partial z}\underline{\mathbf{k}}$ where $\underline{\mathbf{i}}$, $\underline{\mathbf{j}}$, $\underline{\mathbf{k}}$ are unit vectors along x,y,z

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$ rectangular co-ordinates

$$\nabla^2 f = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(r\frac{\partial f}{d\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 f}{\partial\varphi^2} + \frac{\partial^2 f}{\partial z^2}$$ cylindrical co-ordinates

$$\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{dr}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial f}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial\varphi^2}$$ spherical polar co-ordinates

For a vector field $\underline{\mathbf{A}}$, having components along x, y, z of A_x, A_y, A_z

$$\nabla .\ \underline{\mathbf{A}} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \underline{\mathbf{A}} = \begin{vmatrix} \underline{\mathbf{i}} & \underline{\mathbf{j}} & \underline{\mathbf{k}} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ A_x & A_y & A_z \end{vmatrix}$$

$$\nabla(fg) = f\nabla g + g\nabla f$$

$$\nabla.(f\,\underline{A}) = f\nabla.\underline{\mathbf{A}} + \underline{\mathbf{A}}.\nabla f$$

$$\nabla x(f\,\underline{\mathbf{A}}) = f\nabla x\underline{\mathbf{A}} + (\nabla f)x\underline{\mathbf{A}}$$

$$\nabla(\underline{\mathbf{A}}.\underline{\mathbf{B}}) = \underline{\mathbf{A}}x(\nabla x\underline{\mathbf{B}}) + (\underline{\mathbf{A}}.\nabla)\underline{\mathbf{B}} + \underline{\mathbf{B}}x(\nabla x\underline{\mathbf{A}}) + (\underline{\mathbf{B}}.\nabla)\underline{\mathbf{A}}$$

$$\nabla.(\underline{\mathbf{A}}x\underline{\mathbf{B}}) = \underline{\mathbf{B}}.(\nabla x\underline{\mathbf{A}}) - \underline{\mathbf{A}}.(\nabla x\underline{\mathbf{B}})$$

$$\nabla x(\underline{\mathbf{A}}x\underline{\mathbf{B}}) = \underline{\mathbf{A}}(\nabla.\underline{\mathbf{B}}) - \underline{\mathbf{B}}(\nabla.\underline{\mathbf{A}}) + (\underline{\mathbf{B}}.\nabla)\underline{\mathbf{A}} - (\underline{\mathbf{A}}.\nabla)\underline{\mathbf{B}}$$

$$\nabla.(\nabla f) = \nabla^2 f$$

$$\nabla x(\nabla f) = 0$$

$$\nabla.(\nabla x\underline{\mathbf{A}}) = 0$$

$$\nabla x(\nabla x\underline{\mathbf{A}}) = \nabla(\nabla.\underline{\mathbf{A}}) - \nabla^2\,\underline{\mathbf{A}}$$

Gauss' (divergence) theorem: $\int_V (\nabla.\underline{\mathbf{A}})dV = \oint_S \underline{\mathbf{A}}.d\underline{\mathbf{s}}$

Stokes' Theorem : $\int_S (\nabla x\underline{\mathbf{A}})d\underline{\mathbf{s}} = \oint_L \underline{\mathbf{A}}.d\underline{l}$

where:
$\underline{\mathbf{A}}$ is a vector field
dV is a volume element
S_c is a closed surface
V is a volume enclosed
S is a surface
d$\underline{\mathbf{s}}$ is a surface element
L is a loop bounding S
d$\underline{l}$ is a line element

Cysonion Sylfaenol / Fundamental Constants

Quantity	Symbol	Value
pi	π	3.141592
the base of natural logarithms	e	2.718282
Angstrom unit	Å	10^{-10} m
speed of light in vacuuo	c	3×10^{8} ms^{-1}
Planck's constant	h	6.63×10^{-34} Js
Modified Planck’s constant h/2π	$\hbar$	1.05×10^{-34} Js
Electron charge e	e	1.6×10^{-19} C
Acceleration due to gravity	g	9.8 m s^{-2}
Gravitational constant	G	6.67×10^{-11} N m^{2} kg^{-2}
Permittivity of free space	ε_0	8.85×10^{-12} F m^{-1}
Permeability of free space	μ_0	$4\pi \times 10^{-7}$ H m^{-1}
Electron rest mass	m_e	9.1×10^{-31} kg
Energy equivalent of electron rest mass		0.51 MeV
Proton rest mass	M_p	1.6726×10^{-27} kg
Energy equivalent of proton rest mass		938 MeV
Neutron rest mass	M_n	1.6749×10^{-27} kg
Energy equivalent of neutron rest mass		940 MeV
Magnetic moment of electron	μ_e	9.28×10^{-24} J T^{-1}
Magnetic moment of proton	μ_p	1.41×10^{-26} J T^{-1}
Bohr magneton	μ_B	9.27×10^{-24} J T^{-1}
Nuclear magneton	μ_N	5.05×10^{-27} J T^{-1}
Gas constant	R	8.3 J (mol K)$^{-1}$
Avogadro's number	N_A	6×10^{23} mol^{-1}
The Boltzmann constant	k_B	1.38×10^{-23} J K^{-1}
Stefan-Boltzmann constant	σ	5.67×10^{-8} W m^{-2} K^{-4}
Bohr radius	a_0	5.3×10^{-11} m
Atomic mass unit (a.m.u.)	u	1.66×10^{-27} kg
Fine structure constant	α	1/137
1 atmosphere		1.01×10^{5} Pa
1 electron volt (eV)		1.6×10^{-19} J
Mass of Sun		2×10^{30} kg
Mass of Earth		6×10^{24} kg
Radius of Earth		6.38×10^{6} m
Radius of Sun		6.96×10^{8} m
1 AU		1.5×10^{11} m
1 parsec		3.086×10^{16} m