

ADRAN MATHEMATEG / DEPARTMENT OF MATHEMATICS

ARHOLIADAU SEMESTER 2 / SEMESTER 2 EXAMINATIONS

MAI / MAY 2020

MA01020 Mathematics Tutorial

The questions on this paper are written in English.

If you have questions about the paper during the exam, contact the module co-ordinator, Ms. Rosemary Rogers, on ror24@aber.ac.uk.

Amser a ganiateir - 2 awr

Maen rhaid cyflwyno eich atebion erbyn 11:30 (amser y DU).

- Rhoddir marciau llawn am atebion cyflawn i bob cwestiwn.
- Dylai myfyrwyr roi cynnig ar bob cwestiwn **ar bapur**.
- Dylai myfyrwyr **yna** gyflwyno eu hatebion ar safle Blackboard y modiwl hwn erbyn 11.30.

Time allowed - 2 hours

Submission must be completed by 11:30 (UK time).

- Full marks will be given for complete answers to all questions.
- Students should attempt all questions **on paper**.
- Students should **then** submit their answers on the Blackboard site for this module by 11.30.

Notation used

- \mathbb{N} = set of natural (counting) numbers
- \mathbb{Z} = set of integers
- \mathbb{Q} = set of rational numbers
- \mathbb{R} = set of real numbers
- \mathbb{C} = set of complex numbers

Useful formulae**Sequences and series**

- For an arithmetic sequence, the sum of the first n terms is given by $S_n = \frac{1}{2}n(2a + (n - 1)d)$ where n is the number of terms, d is the common difference and a is the first term.
- For a geometric sequence, the sum of the first n terms is given by $S_n = \frac{a(1-r^n)}{1-r}$ where n is the number of terms, r is the common ratio, $r \neq 1$, and a is the first term.
- For a converging geometric sequence, the sum to infinity is given by $S_\infty = \frac{a}{1-r}$.

Trigonometric identities

- $\sin(A + B) = \sin A \cos B + \cos A \sin B$
- $\cos(A + B) = \cos A \cos B - \sin A \sin B$.

Questions

1. Using the factor theorem or otherwise, find all the factors of $f(x) = x^3 - 19x - 30$.

Free text answer – Blackboard will allow you to type an answer.

[5 marks]

2. Which of these is *not* a solution of $2 \cos^2 x - \cos x = 0$, $0 \leq x \leq 2\pi$?

$x = 0$ $x = \frac{\pi}{3}$ $x = \frac{\pi}{2}$ $x = \frac{5\pi}{3}$

[5 marks]

3. Expand and simplify the polar coordinate $\left(2, \frac{\pi}{3}\right)_p$.

$\left(8, \frac{\pi^3}{9}\right)_p$ $\left(8, \frac{\pi}{3}\right)_p$ $\left(6, \frac{\pi^3}{9}\right)_p$ $(8, \pi)_p$ $(6, \pi)_p$

[2 marks]

4. Expand and simplify the complex number $(4 + 3i)(2 - i)$.

Free text answer – Blackboard will allow you to type an answer.

[3 marks]

5. Which is/are the correct solution(s) to $e^{2x} = e^{(1-x^2)}$? (Choose one)

$x = 2$ or $x = -2$
 $x = \ln 2$
 $x = -1 + \sqrt{2}$ or $x = -1 - \sqrt{2}$
 $x = -3$ or $x = 1$

[3 marks]

6. Simplify $2 \log_b 5 - \log_b 10 + \frac{1}{2} \log_b 4$. Express your answer as a single log. You do not need to specify the base when you enter your answer on Blackboard.

Free text answer – Blackboard will allow you to type an answer.

[2 marks]

7. Select the correct domain, codomain and image of the function $f(n) = \frac{1}{n}$, $f : (0, \infty) \rightarrow \mathbb{R}$ from the following choices:

a \mathbb{R} b \mathbb{N} c $(-\infty, \infty)$ d $(0, \infty)$ e $(0, 1)$.

You may use an answer more than once.

Free text answer – Blackboard will allow you to type an answer.

[5 marks]

8. Find all values of x that satisfy $x^2 - 3x > 10$.

Free text answer – Blackboard will allow you to type an answer.

[5 marks]

9. Find the area under the curve $y = 3x^2 - \frac{4}{x^2}$ between $x = 1$ and $x = 2$ and the x -axis.

Free text answer – Blackboard will allow you to type an answer.

[5 marks]

10. Find the n^{th} term and the sum of the first 10 terms of the sequence 2, 7, 12, 17, ...

Free text answer – Blackboard will allow you to type an answer.

[3 marks]

11. Find the sum to infinity of the sequence 36, 12, 4, $\frac{4}{3}$, ...

Free text answer – Blackboard will allow you to type an answer.

[2 marks]

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12. The equation $x^2 + kx + m = 0$ has roots α and β . Use Vieta's Theorem to find an equation with roots $\frac{1}{\alpha}$ and $\frac{1}{\beta}$, leaving your answer in terms of k and m .
You can enter x^2 as x^2 on Blackboard.

Free text answer – Blackboard will allow you to type an answer.

[5 marks]

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13. Which of these functions are one-one? Choose as many as you wish. In all cases the domain and codomain are \mathbb{R} .

- $f(x) = x^2$
 $g(x) = x^3$
 $h(x) = 3 - x$
 $j(x) = e^x$
 $k(x) = \cos x$

[5 marks]