PRIFYSGOL ABERYSTWYTH UNIVERSITY

## ADRAN MATHEMATEG / DEPARTMENT OF MATHEMATICS

## ARHOLIADAU SEMESTER 2 / SEMESTER 2 EXAMINATIONS

## MAI / MAY 2020

# MA11110 - Mathematical Analysis

The questions on this paper are written in English.

If you have questions about the paper during the exam, contact the module co-ordinator, Dr Rob Douglas, on rsd@aber.ac.uk.

**Amser a ganiateir - 2 awr**
*Maen rhaid cyflwyno eich atebion erbyn 11:30 (amser y DU).*

- Rhoddir marciau llawn am atebion cyflawn i bob cwestiwn.
- Dylai myfyrwyr roi cynnig ar bob cwestiwn **ar bapur**.
- Dylai myfyrwyr **yna** gyflwyno eu hatebion ar safle Blackboard y modiwl hwn.

**Time allowed - 2 hours**
*Submission must be completed by 11:30 (UK time).*

- Full marks will be given for complete answers to all questions.
- Students should attempt all questions **on paper**.
- Students should **then** submit their answers on the Blackboard site for this module.

## Questions

**1**. Classify the following subsets of $\mathbb{R}$ as bounded or unbounded. If the set is bounded, write down the supremum and infimum.

(a) $\{x : |x+3| < 4\}$; (b) $\{\exp(x) : x \geq 4\}$; (c) $\{x : x^2 = 4\}$;

(d) $\{1/n^4 : n \in \mathbb{N} \text{ is a prime number}\}$; (e) $\bigcap_{n=1}^{\infty} \left(7 - \frac{1}{n}, 9 + \frac{1}{n}\right)$.

[13 marks]

*Free text answer – Blackboard will allow you to type an answer.*

**2**. Let $A, B$ be non-empty bounded subsets of $\mathbb{R}$. For $\alpha, \beta \in \mathbb{R}$, define

$$\alpha A + \beta B = \{\alpha x + \beta y : x \in A, y \in B\}.$$

If $\alpha \geq 0$, $\beta < 0$, what is the value of $\sup(\alpha A + \beta B)$, where $\sup C, \inf C$ denote respectively the supremum and infimum of the set $C$?

- ◯ $\alpha \sup A + \beta \sup B$
- ◯ $\alpha \sup A + \beta \inf B$
- ◯ $\alpha \sup A - \beta \inf B$
- ◯ $\alpha \inf A + \beta \inf B$

[2 marks]

**3**. Calculate the following limits:

(a) $\lim_{n\to\infty} \frac{12n^6 - 2n^5 + 6}{16n^6 + 3n^3 + 4}$; (b) $\lim_{n\to\infty} \sqrt{n^2 + 8n} - n$; (c) $\lim_{n\to\infty} \frac{n^2 2^{n+1}}{5^n}$.

[2,3,3 marks]

*Free text answer – Blackboard will allow you to type an answer.*

**4**. Define a sequence $(a_n)_{n=1}^{\infty}$ of real numbers recursively by

$$a_1 = 2,\ a_{n+1} = (a_n + 1)/2 \text{ for } n \in \mathbb{N}.$$

(a) Calculate $a_2$ and $a_3$. [2 marks]

*Free text answer – Blackboard will allow you to type an answer.*

(b) Classify the sequence $(a_n)_{n=1}^{\infty}$ as either monotone increasing, monotone decreasing, or not monotonic. [2 marks]

*Free text answer – Blackboard will allow you to type an answer.*

(c) Determine $\lim_{n\to\infty} a_n$. [2 marks]

*Free text answer – Blackboard will allow you to type an answer.*

**5**. Classify the following series as either convergent or non-convergent. If the series is convergent, find its sum.

$$\text{(a) } \sum_{n=0}^{\infty} \frac{6^{n+2}}{7^n};\ \text{(b) } \sum_{n=0}^{\infty} \frac{(-4)^n}{26}.$$

[2,2 marks]

*Free text answer – Blackboard will allow you to type an answer.*

**6**. Classify the following series as either convergent or non-convergent. State the test which you use (from Vanishing Test, Comparison Test, Ratio Test and Alternating Series Test).

$$\text{(a) } \sum_{n=1}^{\infty} \frac{3n+1}{n^3};\ \text{(b) } \sum_{n=1}^{\infty} \frac{(3n)!}{n!(2n)!};\ \text{(c) } \sum_{n=1}^{\infty} \frac{n+3}{6n+1};\ \text{(d) } \sum_{n=1}^{\infty} \frac{7n^2+4}{5n^3+2};\ \text{(e) } \sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n+4}.$$

[2,2,2,2,2 marks]

*Free text answer – Blackboard will allow you to type an answer.*

**7**. Suppose that $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are absolutely convergent series.

(a) Does $\sum_{n=1}^{\infty} a_n b_n$ have to converge? (Yes/No) [2 marks]

*Free text answer – Blackboard will allow you to type an answer.*

(b) Which two tests from the following list would you use?

- Vanishing Test
- Comparison Test
- Ratio Test
- Alternating Series Test

[2 marks]

*Free text answer – Blackboard will allow you to type an answer.*

**8**. Evaluate the following limits:

$$\text{(a) } \lim_{x\to 0} \frac{\cos(4x)-1}{2x^2}; \text{ (b) } \lim_{x\to 0} \frac{\exp(3x)-x-1}{4x}.$$

[3,2 marks]

*Free text answer – Blackboard will allow you to type an answer.*

**9**. Suppose $f : \mathbb{R} \to \mathbb{R}$ is continuous and satisfies

$$\lim_{x\to -\infty} f(x) = \lim_{x\to \infty} f(x) = \infty.$$

(a) Does $f$ have a maximum value? (Yes/No) [1 mark]

*Free text answer – Blackboard will allow you to type an answer.*

(b) Does $f$ have a minimum value? (Yes/No) [2 marks]

*Free text answer – Blackboard will allow you to type an answer.*

**10**. How many real roots does the equation $3x^7 + 17x^3 + 4x + 23 = 0$ have? [2 marks]

*Free text answer – Blackboard will allow you to type an answer.*

**11**. Let $g : [0, 3] \to [0, 3]$ be continuous.

(a) Is $g$ guaranteed to have a fixed point, that is does there exist $x_0 \in [0, 3]$ such that $g(x_0) = x_0$? (Yes/No) [1 mark]

*Free text answer – Blackboard will allow you to type an answer.*

(b) Which theorem from the following list would you apply?

- Intermediate Value Theorem
- Rolle's Theorem
- Lagrange Mean Value Theorem
- The Old Cobbler's Theorem [2 marks]

*Free text answer – Blackboard will allow you to type an answer.*