

ADRAN MATHEMATEG / DEPARTMENT OF MATHEMATICS

ARHOLIADAU SEMESTER 2 / SEMESTER 2 EXAMINATIONS

MAI / MAY 2020

MP11010 - Further Algebra and Calculus

The questions on this paper are written in English.

If you have questions about the paper during the exam, contact the module co-ordinator, Prof. Simon Cox, on sxc@aber.ac.uk.

Amser a ganiateir - 2 awr

Maen rhaid cyflwyno eich atebion erbyn 11:30 (amser y DU).

- Rhoddir marciau llawn am atebion cyflawn i bob cwestiwn.
- Dylai myfyrwyr roi cynnig ar bob cwestiwn **ar bapur**.
- Dylai myfyrwyr **yna** gyflwyno eu hatebion ar safle Blackboard y modiwl hwn erbyn 11.30.

Time allowed - 2 hours

Submission must be completed by 11:30 (UK time).

- Full marks will be given for complete answers to all questions.
- Students should attempt all questions **on paper**.
- Students should **then** submit their answers on the Blackboard site for this module by 11.30.

Questions

1. Given

$$\underline{\underline{A}} = \begin{pmatrix} 1 & 3 \\ -2 & 4 \\ -5 & 2 \end{pmatrix}, \quad \underline{\underline{B}} = \begin{pmatrix} 0 & -1 \\ 2 & 3 \end{pmatrix}, \quad \text{and} \quad \underline{\underline{C}} = \begin{pmatrix} 1 & 2 & -1 \\ 1 & 0 & 4 \end{pmatrix},$$

determine which of the following matrices are defined and compute them when possible.

(a) Is $\underline{\underline{A}}^T + \underline{\underline{B}}$ defined? If it is, give the first row of the resulting matrix.

Free text answer – Blackboard will allow you to type an answer.

(b) Is $\underline{\underline{B}}\underline{\underline{C}} + 2\underline{\underline{A}}^T$ defined? If it is, give the first row of the resulting matrix.

Free text answer – Blackboard will allow you to type an answer.

(c) Is $\underline{\underline{C}}\underline{\underline{B}}^{-1}$ defined? If it is, give the first row of the resulting matrix.

Free text answer – Blackboard will allow you to type an answer.

[5 marks]

2. Calculate the determinant of the matrix

$$\underline{\underline{A}} = \begin{pmatrix} 2 & 4 \\ -1 & -3 \end{pmatrix}.$$

Free text answer – Blackboard will allow you to type an answer.

[2 marks]

3. Calculate the determinant of the matrix

$$\underline{\underline{B}} = \begin{pmatrix} -3 & 1 & 0 \\ 2 & 2 & 5 \\ -3 & 4 & 6 \end{pmatrix}.$$

Free text answer – Blackboard will allow you to type an answer.

[3 marks]

4. Given

$$\underline{\underline{A}} = \begin{pmatrix} 5 & 2 \\ 7 & 3 \end{pmatrix} \quad \text{and} \quad \underline{\underline{b}} = \begin{pmatrix} -1 \\ 3 \end{pmatrix},$$

solve the system $\underline{\underline{A}}\underline{\underline{x}} = \underline{\underline{b}}$ to find $\underline{\underline{x}}$ by inverting the matrix $\underline{\underline{A}}$. Enter your solution for $\underline{\underline{x}}$ in Blackboard.

(NB: You can type the vector $\underline{\underline{x}}$ in the form (x_1, x_2) .)

Free text answer – Blackboard will allow you to type an answer.

[5 marks]

5. Use Gaussian elimination to reduce the matrix

$$\underline{\underline{A}} = \begin{pmatrix} 4 & 0 & 1 \\ 3 & -1 & 0 \\ 1 & -2 & 1 \end{pmatrix}$$

to (row) echelon form.

List the row operations that you used in the form $r1 = r1 + 2 r3$, $r1 < - > r2$, etc. You do not need to enter the resulting matrix into Blackboard.

Free text answer – Blackboard will allow you to type an answer.

[5 marks]

6. Consider the matrix

$$\underline{\underline{A}} = \begin{pmatrix} 5 & 7 \\ 4 & 2 \end{pmatrix}.$$

Give the eigenvalues of $\underline{\underline{A}}$:

Free text answer – Blackboard will allow you to type an answer.

[2 marks]

Give unit eigenvectors for $\underline{\underline{A}}$:

(NB: You can type the vectors in the form (x,y) and use $\text{sqrt}(x)$ for a square-root.)

Free text answer – Blackboard will allow you to type an answer.

[3 marks]

7. Let $f(x, y) = x^3 + 3x^2y + 3y^2 - 18y$.

(a) Enter the values of $\frac{\partial f}{\partial y}$ at the point $(1, 1)$ and of $\frac{\partial^2 f}{\partial x^2}$ at the point $(2, 3)$ into Blackboard.

Free text answer – Blackboard will allow you to type an answer.

[3 marks]

(b) Find the critical points of $f(x, y)$ and determine their nature. Enter into Blackboard the critical points (in the form (x, y)) and state whether each is a maximum, minimum, or saddle point.

Free text answer – Blackboard will allow you to type an answer.

[5 marks]

8. Evaluate the double integral

$$\iint_H (x^2 + y^2) \, dA,$$

where H is the rectangle $0 \leq x \leq 2, 0 \leq y \leq 3$.

Free text answer – Blackboard will allow you to type an answer.

[4 marks]

9. $f(x, y)$ is a function of the independent variables x and y , but may be considered as a function of the independent variables u and v , which are related to x and y by the equations $x = 4uv$ and $y = (u^2 - v^2)$.

Using the chain rule, evaluate f_u and f_v and show that

$$uf_u + vf_v = axf_x + byf_y \quad \text{and} \quad vf_u - uf_v = cyf_x + dx f_y$$

for particular values of a, b, c and d .

Enter the parameters a, b, c and d into Blackboard.

Free text answer – Blackboard will allow you to type an answer.

[6 marks]

10. Determine the volume lying beneath the plane $z = 6x + 9y + 18$ and above the triangle with vertices at $(0, 0, 0)$, $(1, 0, 0)$ and $(0, 2, 0)$.

Specify on Blackboard the limits you use for the integrals, followed by the answer. For example, " $x = 1$ to $6y$, $y = 2$ to 4 , Volume = 99 ".

Free text answer – Blackboard will allow you to type an answer.

[7 marks]