PRIFYSGOL ABERYSTWYTH UNIVERSITY

ARHOLIADAU EXAMINATIONS

SEMESTER TWO EXAMINATION 2026

ABERYSTWYTH BUSINESS SCHOOL

AB31320 – INVESTMENTS AND FINANCIAL INSTRUMENTS

Time allowed: THREE HOURS

Answer all questions in SECTION A

AND

Answer TWO questions in SECTION B. QUESTION B1 IS COMPULSORY

AND

Answer TWO questions in SECTION C

Selected formulae, present value factor tables and standard normal distribution tables are attached to the back of this examination paper.

Total marks available**: 100**

This paper contributes **70%** of the overall module mark.

Answer booklets and graph sheets will be provided.

CALCULATORS ARE PERMITTED (Casio FX991, FX85 or FX83 models only)

This is an UNSEEN BOOK examination.

MAE MODD I FYFYRWYR GYFLWYNO ATEBION I'R PAPUR HWN NAILL AI YN Y GYMRAEG NEU'R SAESNEG
STUDENTS MAY SUBMIT ANSWERS TO THIS PAPER IN EITHER WELSH OR ENGLISH
Once seated, students may complete the front cover of the answer book(s) and the attendance slip ONLY.

DO NOT TURN THE EXAMINATION QUESTION PAPER OVER UNTIL INSTRUCTED TO DO SO.

Section A

This is a multiple-choice examination containing 20 questions (20 marks). There are five possible answers to each question, but only one of them is correct. **1 mark is awarded** for every correct answer; **No marks are added or deducted** for incorrect or blank answers.

Answer **ALL** questions. You **must** clearly write the question number and your answer to the question in your examination answer booklet.

Question A1

Most corporate bonds are traded

A. by the issuing corporation.
B. on a formal exchange operated by the London Stock Exchange.
C. on a formal exchange operated by the New York Stock Exchange.
D. over the counter by bond dealers linked by a quotation system.
E. on a formal exchange operated by the Tokyo Stock Exchange.

Question A2

Which of the following bonds has the longest duration?

A. An 8-year maturity, 0% coupon bond
B. A 10-year maturity, 5% coupon bond
C. An 8-year maturity, 5% coupon bond
D. A 10-year maturity, 0% coupon bond
E. A 10-year maturity, 3% coupon bond

Question A3

When interest rates decline, the duration of a bond selling at a premium

A. decreases.
B. increases.
C. remains the same.
D. increases at first, then decreases.
E. decreases at first, then increases.

Turn Overleaf

Question A4

The current yield on a bond is equal to

A. the yield to maturity.
B. the internal rate of return.
C. annual interest payment divided by the current market price.
D. annual interest payment divided by the par value.
E. annual coupon payment divided by the par value.

Question A5

Consider two bonds, A and B. Both bonds presently are selling at their par value of £1,000. Each pays interest of £85 annually. Bond A will mature in 12 years, while bond B will mature in 25 years. If the yields to maturity on the two bonds change from 9% to 12%,

A. both bonds will decrease in value, but bond A will decrease more than bond B.
B. both bonds will decrease in value, but bond B will decrease more than bond A.
C. both bonds will increase in value, but bond A will increase more than bond B.
D. both bonds will increase in value, but bond B will increase more than bond A.
E. both bonds will decrease in value equally.

Question A6

A coupon bond pays annual interest, has a par value of £100, matures in four years, has a coupon rate of 10%, and has a yield to maturity rate of 12%. The current yield of this bond is

A. 12%
B. 10%
C. 10.45%
D. 10.65%
E. 10.95%

Question A7

Holding other factors constant, which one of the following bonds has the smallest price volatility?

A. 8-year, 0% coupon bond
B. 8-year, 5% coupon bond
C. 8-year, 10% coupon bond
D. 8-year, 15% coupon bond
E. Cannot tell from the information given

Turn Overleaf

Question A8

A zero-coupon bond has a yield to maturity of 9% and a par value of £100. If the bond matures in eight years and pays coupon twice a year, the bond should sell for a price of _____ today.

A. £42.24
B. £51.32
C. £41.87
D. £49.45
E. £48.35

Question A9

The two components of interest-rate risks are:

A. price risk and default risk.
B. call risk and price risk.
C. reinvestment risk and systematic risk.
D. call risk and reinvestment risk.
E. price risk and reinvestment risk.

Question A10

Altman's Z scores are assigned based on a firm's financial characteristics and are used to predict

A. the probability of a bond issue being called.
B. the likelihood of a firm becoming a takeover target.
C. required coupon rates for new bond issues.
D. bankruptcy risk.
E. None of the options are correct.

Question A11

If the stock price increases, the price of a put option on that stock _____, and that of a call option _____.

A. increase; increase
B. decrease; increase
C. increase; decrease
D. decrease; decrease
E. does not change; does not change

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Question A12

All the inputs in the Black-Scholes option pricing model are directly observable except

A. the price of the underlying security.
B. the option exercise price.
C. the variance of returns of the underlying asset return.
D. the risk-free rate of interest.
E. the time to expiration.

Question A13

A call option on a stock is said to be out of the money if

A. the exercise price is less than the stock price.
B. the exercise price is higher than the stock price.
C. the exercise price is equal to the stock price.
D. the price of the put is higher than the price of the call.
E. the price of the call is higher than the price of the put.

Question A14

An investor purchases one £60 call contract and one equal amount of put contract on the same stock. The call premium is £5 and the put premium is £3. The strategy is called

A. a long straddle
B. a short straddle
C. a horizontal straddle
D. a covered call
E. a protective put

Question A15

An investor purchases one £200 put contract for a put premium of £6. What is the maximum payoff and profit that the investor could gain from this strategy?

A. £206, £6
B. £194, 206
C. £0, £0
D. £200, £194
E. £206, £194

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Question A16

The put-call parity theorem

A. represents the proper relationship between put and call prices.
B. allows for arbitrage opportunities if violated.
C. may be violated by small amounts, but not enough to earn arbitrage profits once transaction costs are considered.
D. only applies to European options, which can only be exercised at expiration.
E. All the options are correct.

Question A17

Credit risk in the swap market

A. is equal to the total value of the payments that the floating rate payer was obligated to make.
B. is limited to the difference between the values of the fixed rate and the floating rate obligates.
C. is extensive.
D. is extensive and equal to the total value of the payments that the floating rate payer was obligated to make.
E. None of the options are correct.

Question A18

Who guarantees that a futures contract will be fulfilled?

A. The buyer
B. The seller
C. The broker
D. The clearinghouse
E. Nobody

Question A19

In a futures contract, the futures price is

A. determined by the buyer and the seller when the delivery of the commodity takes place.
B. determined by the futures exchange.
C. determined by the buyer and the seller when they initiate the contract.
D. determined independently by the provider of the underlying asset.
E. None of the options are correct.

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Question A20

Which one of the following statements is true?

A. A margin deposit can only be met with cash.
B. All futures contracts require the same margin deposit.
C. The maintenance margin is the amount of money you post with your broker when you buy or sell a futures contract.
D. If the value of the margin account falls below the maintenance margin requirement, the holder of the contract will receive a margin call.
E. The maintenance margin is set by the producer of the underlying asset.

END OF THE SECTION A

SECTION B FOLLOWS

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Section B

Each question in section B carries 30 marks.
Question B1 is **compulsory** and
Answer **either** Question B2 **or** B3
Answer **ALL** parts of your selected questions.
Show the details and steps followed to arrive at your answers.

Question B1 – compulsory (30 marks)

Given a two-asset economy, Table 1 below shows the expected returns and risks of the two assets:

Table 1

Asset	**Gil**	**Equ**
Expected Return (r)	5%	12%
Standard Deviation of Returns (σ)	11%	21%

and with the correlation of expected returns between the two assets being -0.2 and the risk-free rate being 4%.

a) Under the minimum-variance portfolio theory, find the expected return and risk of the minimum variance portfolio.

[6 marks]

b) Under this two-asset economy assumption, find the expected return and risk of the optimal risky portfolio.

[6 marks]

c) From the results of part **b),** deduce the capital market line (CML) equation and draw the CML with the implied Y axis (expected return) against the X axis (total risk).

[4 marks]

d) Plot the results from part **a) and b)** on the same graph deduced in part **c)** to show the estimated efficiency frontier.

[3 marks]

e) Given an investor with a risk-aversion coefficient of A=4, estimate the expected return and risk of the investor's optimal complete portfolio.

[4 marks]

f) Determine the utility associated with the above optimal complete portfolio.

[2 marks]

g) Plot the results from parts **e)** and **f)** on the same graph deduced in part **d)** to show the estimated optimal complete portfolio and the utility indifference curve. Comment on the final graph from parts **d)** and **g)**

[5 marks]

[Total 30 marks]

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Answer **either** Question B2 **or** B3

Question B2 (30 marks)

Answer **either** Question B2 **or** B3

Table 2 below shows the expected interest rates for the following 8 years:

Table 2

Year	Forward Interest Rate
1	5.10%
2	5.80%
3	7.20%
4	8.50%
5	9.00%
6	9.90%
7	10.50%
8	11.05%

EFG plc issued a 5-year corporate note with a 5% annual coupon rate and a £100 principal value. The note makes an annual coupon payment.

a) Use the information provided in Table 2 to predict the YTM to a note with maturity term of 5 years.

[2 marks]

b) Use the YTM from part **a)** to calculate the price, the Macaulay duration, and the modified duration for the note.

[8 marks]

c) Use the modified duration from part **b)** to estimate the changes in the note's current value if the YTM increases or decreases by 2% annually. Comment on the relationship between the debt security price and the YTM rate.

[5 marks]

d) Calculate the modified convexity of the note and proceed to use the convexity to improve the estimation of the note price changes from part **c).**

[10 marks]

e) Present the numerical results to demonstrate the accuracy of the note price change estimations derived in parts **c)** and **d)**. Comment on how closely the results align with the earlier estimates.

[5 marks]

[Total 30 marks]

Turn Overleaf

Question B3 (30 marks)

Assume you are financial analyst who is responsible for a client who is considering using financial options to manage risk in a highly volatile market environment. The client is evaluating investment strategies involving Celtic plc, a US firm whose share price has recently experienced sharp fluctuations due to the ongoing geopolitical uncertainty.

Celtic plc's share price is currently £50. The price is estimated to move up or down by 20% in each of the next two semi-annual periods. To manage exposure to this volatility, the client is considering two derivative instruments: a call option with an exercise price of £55 or a put option with an exercise price of £45. Both options mature in one year.

The risk-free interest rate is 4% per annum, compounded continuously.

a) Determine the probability of the share price's movement being up or down.

[2 marks]

b) Use the Binomial model to determine the value of the one-year call and put option price today.

[8 marks]

c) Repeat part **c)** using the Black Scholes model.

[10 marks]

d) From the result of either part **b)** or part **c)**, use graphs to illustrate the payoff and profit to a holder of the call and put options at expiration.

[6 marks]

e) Comment on the outcomes of each strategy to help the client to decide.

[4 marks]

[Total 30 marks]

END OF THE SECTION B

SECTION C FOLLOWS

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Section C

Each question in section C carries 10 marks.
Answer **ANY TWO** questions.

Question C1

State the key assumptions underlying the Capital Asset Pricing Model (CAPM). Discuss the limitations of these assumptions and the implications when investors rely on the CAPM to make their investment decisions.

[10 marks]

Question C2

The Efficient Market Hypothesis (EMH) proposes weak, semi-strong and strong forms to explain how different types of information are incorporated into asset prices. However, numerous market anomalies challenge the validity of each form. Using appropriate examples of anomalies, discuss how they contradict the predictions of EMH and how these contradictions affect investor behaviour.

[10 marks]

Question C3

Evaluate the different theories of the term structure of interest rate. Explain how future short-term and long-term interest rates are determined and use diagrams where appropriate to support your answer.

[10 marks]

Question C4

Considering the relationship between bond prices and yields to maturity (YTM), discuss how changes in market interest rates impact bond prices. Use a numerical example, illustrate how the price-yield relationship differs for bonds with different coupon rates and maturities.

[10 marks]

END OF PAPER

SELECTED FORMULAE, PRESENT VALUE TABLES, AND STANDARD NORMAL DISTRIBUTION TABLES FOLLOW

Turn Overleaf

SELECTED FORMULAE

Arithmetic mean: $\frac{r_1+r_2+r_3+\cdots+r_n}{n}$

Geometric mean: $\sqrt[n]{(1+r_1)\times(1+r_2)\times\cdots\times(1+r_n)}-1$

HPR: $r_{j,t}=\frac{p_{j,t}-p_{j,t-1}+d_{j,t}}{p_{j,t-1}}$

Logarithmic return: ln(1+HPR)

Probability weighted return: $E(X)=P_r(X_1)X_1+P_r(X_2)X_2+\cdots+P_r(X_N)X_N$

Variance: $\sigma^2=[\sum(X-\mu)^2]/N$

Probability weighted variance:
$\sigma^2=P_r(X_1)(X_1-\mu)^2+P_r(X_2)(X_2-\mu)^2+\cdots+P_r(X_N)(X_N-\mu)^2$

Standard deviation: $\sigma=\sqrt{\sigma^2}$

Covariance: $\sigma_{1,2}=\frac{(X_1-\mu_1)\times(X_2-\mu_2)}{N}$

Correlation: $\rho_{1,2}=\frac{\sigma_{1,2}}{\sigma_1\times\sigma_2}$

$W_{\min(1)}=\frac{\sigma_2^2-\sigma_1\sigma_2\rho_{1,2}}{\sigma_1^2+\sigma_2^2-2\sigma_1\sigma_2\rho_{1,2}}$ or $W_{\min(1)}=\frac{\sigma_2^2-\sigma_{1,2}}{\sigma_1^2+\sigma_2^2-2\sigma_{1,2}}$

$W_{\min(2)}=1-W_{\min(1)}$

$W_{\max(1)}=\frac{(r_1-r_f)\sigma_2^2-(r_2-r_f)\sigma_1\sigma_2\rho_{1,2}}{(r_1-r_f)\sigma_2^2+(r_2-r_f)\sigma_1^2-[(r_1-r_f)+(r_2-r_f)]\sigma_1\sigma_2\rho_{1,2}}$ or,

$$W_{\max(1)}=\frac{(r_1-r_f)\sigma_2^2-(r_2-r_f)\sigma_{1,2}}{(r_1-r_f)\sigma_2^2+(r_2-r_f)\sigma_1^2-[(r_1-r_f)+(r_2-r_f)]\sigma_{1,2}}$$

$W_{\max(2)}=1-W_{\max(1)}$

$E(r_p)=w_1r_1+w_2r_2$

$$\sigma_p=\sqrt{w_1^2\sigma_1^2+w_2^2\sigma_2^2+2w_1w_2\rho_{1,2}\sigma_1\sigma_2}$$

$\sigma_p=\sqrt{22\mu_p^2-4.8\mu_p+0.33}$

Sharp ratio: S= $\frac{r_m - r_f}{\sigma_m}$

CAL/CML $\mu_p = r_f + \gamma\sigma_p, \gamma = S = \frac{r_m - r_f}{\sigma_m},$

Or $\gamma = 2\sqrt{\frac{-(ar_f^2 + br_f + c)}{b^2 - 4ac}}$

Utility: $U = E(r) - 1/2\ A\sigma^2$

Weight of portfolio to reach a maximum utility: $Y* = \frac{E(r_p) - r_f}{A\sigma_p^2}$

$$\beta_j = \frac{r_j - r_f}{r_M - r_f} = \frac{cov(r_M, r_{j)}}{\sigma_M^2}$$

$$\text{CAPM: } r_j = r_f + \beta_j(r_M - r_f)$$

APT: $r_i = \alpha_i + b_i F + \varepsilon_i, F = (r_p - r_f)$

$$R_i = a_i + b_{i1}F_1 + b_{i2}F_2 + \varepsilon_i$$

$$(1 + s_n)^n = (1 + f_1)(1 + f_2)\cdots(1 + f_{n-1})(1 + f_n)$$

$$s_n = \sqrt[n]{(1 + f_1)(1 + f_2)\cdots(1 + f_{n-1})(1 + f_n)} - 1$$

$$f_n = \frac{(1 + s_n)^n}{(1 + f_1)(1 + f_2)\cdots(1 + f_{n-1})} - 1$$

$$f_n = \frac{(1 + s_n)^n}{(1 + s_{n-1})^{n-1}} - 1$$

$$PV_{Bond} = \frac{Coupon}{YTM} \times \left[1 - \frac{1}{(1 + YTM)^t}\right] + \frac{par}{(1 + YTM)^t}$$

$$D = \frac{1}{P} \times \sum_{t=1}^{t} (t \times PV_t)$$

$$MD = D/(1 + YTM)$$

$$conv = \frac{1}{P} \times \frac{1}{(1 + YTM)^2} \times \sum_{t=1}^{t} t(1 + t) \times \frac{CF_t}{(1 + YTM)^t}$$

$$\frac{\triangle P}{P} = -MD \times \triangle YTM$$

$$\frac{\triangle P}{P} = -MD \times \triangle YTM + conv \times \frac{\triangle YTM^2}{2}$$

$$u = e^{\sigma \times \sqrt{\Delta t}}$$

$$d = e^{-\sigma \times \sqrt{\Delta t}}$$

$$\sigma = \ln(u) / \sqrt{\Delta t}$$

$$Delta = \frac{C_u - C_d}{S_u - S_d}$$

$$u_p = \frac{1 + r_f - d}{u - d}$$

$$d_p = 1 - u_p$$

$$C = \frac{C_u u_p + C_d d_p}{1 + r}$$

$$C = \frac{p^2 C_{uu} + 2p(1-p) C_{ud} + (1-p)^2 C_{dd}}{(1+r)^2}$$

$$C = \frac{p^3 C_{u^3} + 3p^2(1-p) C_{u^2 d} + 3p(1-p)^2 C_{ud^2} + (1-p)^3 C_{d^3}}{(1+r)^3}$$

$$C = S_0 \times N(d_1) - X e^{-rT} \times N(d_2)$$

$$d_1 = \frac{\ln\left(\frac{S_0}{X}\right) + (r + 0.5 \times \sigma^2) \times T}{\sigma \sqrt{T}}$$

$$d_2 = d_1 - \sigma \sqrt{T}$$

$$P = C + PV(X) - S_0$$

Table 1: Present value of 1 unit received n periods in the future assuming a constant discount rate of r per period

n \ r	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	11%	12%	13%	14%	15%	16%	17%	18%	19%	20%	21%	22%	23%	24%	25%	30%	35%	40%
1	0.990	0.980	0.971	0.962	0.952	0.943	0.935	0.926	0.917	0.909	0.901	0.893	0.885	0.877	0.870	0.862	0.855	0.847	0.840	0.833	0.826	0.820	0.813	0.806	0.800	0.769	0.741	0.714
2	0.980	0.961	0.943	0.925	0.907	0.890	0.873	0.857	0.842	0.826	0.812	0.797	0.783	0.769	0.756	0.743	0.731	0.718	0.706	0.694	0.683	0.672	0.661	0.650	0.640	0.592	0.549	0.510
3	0.971	0.942	0.915	0.889	0.864	0.840	0.816	0.794	0.772	0.751	0.731	0.712	0.693	0.675	0.658	0.641	0.624	0.609	0.593	0.579	0.564	0.551	0.537	0.524	0.512	0.455	0.406	0.364
4	0.961	0.924	0.888	0.855	0.823	0.792	0.763	0.735	0.708	0.683	0.659	0.636	0.613	0.592	0.572	0.552	0.534	0.516	0.499	0.482	0.467	0.451	0.437	0.423	0.410	0.350	0.301	0.260
5	0.951	0.906	0.863	0.822	0.784	0.747	0.713	0.681	0.650	0.621	0.593	0.567	0.543	0.519	0.497	0.476	0.456	0.437	0.419	0.402	0.386	0.370	0.355	0.341	0.328	0.269	0.223	0.186
6	0.942	0.888	0.837	0.790	0.746	0.705	0.666	0.630	0.596	0.564	0.535	0.507	0.480	0.456	0.432	0.410	0.390	0.370	0.352	0.335	0.319	0.303	0.289	0.275	0.262	0.207	0.165	0.133
7	0.933	0.871	0.813	0.760	0.711	0.665	0.623	0.583	0.547	0.513	0.482	0.452	0.425	0.400	0.376	0.354	0.333	0.314	0.296	0.279	0.263	0.249	0.235	0.222	0.210	0.159	0.122	0.095
8	0.923	0.853	0.789	0.731	0.677	0.627	0.582	0.540	0.502	0.467	0.434	0.404	0.376	0.351	0.327	0.305	0.285	0.266	0.249	0.233	0.218	0.204	0.191	0.179	0.168	0.123	0.091	0.068
9	0.914	0.837	0.766	0.703	0.645	0.592	0.544	0.500	0.460	0.424	0.391	0.361	0.333	0.308	0.284	0.263	0.243	0.225	0.209	0.194	0.180	0.167	0.155	0.144	0.134	0.094	0.067	0.048
10	0.905	0.820	0.744	0.676	0.614	0.558	0.508	0.463	0.422	0.386	0.352	0.322	0.295	0.270	0.247	0.227	0.208	0.191	0.176	0.162	0.149	0.137	0.126	0.116	0.107	0.073	0.050	0.035
11	0.896	0.804	0.722	0.650	0.585	0.527	0.475	0.429	0.388	0.350	0.317	0.287	0.261	0.237	0.215	0.195	0.178	0.162	0.148	0.135	0.123	0.112	0.103	0.094	0.086	0.056	0.037	0.025
12	0.887	0.788	0.701	0.625	0.557	0.497	0.444	0.397	0.356	0.319	0.286	0.257	0.231	0.208	0.187	0.168	0.152	0.137	0.124	0.112	0.102	0.092	0.083	0.076	0.069	0.043	0.027	0.018
13	0.879	0.773	0.681	0.601	0.530	0.469	0.415	0.368	0.326	0.290	0.258	0.229	0.204	0.182	0.163	0.145	0.130	0.116	0.104	0.093	0.084	0.075	0.068	0.061	0.055	0.033	0.020	0.013
14	0.870	0.758	0.661	0.577	0.505	0.442	0.388	0.340	0.299	0.263	0.232	0.205	0.181	0.160	0.141	0.125	0.111	0.099	0.088	0.078	0.069	0.062	0.055	0.049	0.044	0.025	0.015	0.009
15	0.861	0.743	0.642	0.555	0.481	0.417	0.362	0.315	0.275	0.239	0.209	0.183	0.160	0.140	0.123	0.108	0.095	0.084	0.074	0.065	0.057	0.051	0.045	0.040	0.035	0.020	0.011	0.006
16	0.853	0.728	0.623	0.534	0.458	0.394	0.339	0.292	0.252	0.218	0.188	0.163	0.141	0.123	0.107	0.093	0.081	0.071	0.062	0.054	0.047	0.042	0.036	0.032	0.028	0.015	0.008	0.005
17	0.844	0.714	0.605	0.513	0.436	0.371	0.317	0.270	0.231	0.198	0.170	0.146	0.125	0.108	0.093	0.080	0.069	0.060	0.052	0.045	0.039	0.034	0.030	0.026	0.023	0.012	0.006	0.003
18	0.836	0.700	0.587	0.494	0.416	0.350	0.296	0.250	0.212	0.180	0.153	0.130	0.111	0.095	0.081	0.069	0.059	0.051	0.044	0.038	0.032	0.028	0.024	0.021	0.018	0.009	0.005	0.002
19	0.828	0.686	0.570	0.475	0.396	0.331	0.277	0.232	0.194	0.164	0.138	0.116	0.098	0.083	0.070	0.060	0.051	0.043	0.037	0.031	0.027	0.023	0.020	0.017	0.014	0.007	0.003	0.002
20	0.820	0.673	0.554	0.456	0.377	0.312	0.258	0.215	0.178	0.149	0.124	0.104	0.087	0.073	0.061	0.051	0.043	0.037	0.031	0.026	0.022	0.019	0.016	0.014	0.012	0.005	0.002	0.001
21	0.811	0.660	0.538	0.439	0.359	0.294	0.242	0.199	0.164	0.135	0.112	0.093	0.077	0.064	0.053	0.044	0.037	0.031	0.026	0.022	0.018	0.015	0.013	0.011	0.009	0.004	0.002	0.001
22	0.803	0.647	0.522	0.422	0.342	0.278	0.226	0.184	0.150	0.123	0.101	0.083	0.068	0.056	0.046	0.038	0.032	0.026	0.022	0.018	0.015	0.013	0.011	0.009	0.007	0.003	0.001	0.001
23	0.795	0.634	0.507	0.406	0.326	0.262	0.211	0.170	0.138	0.112	0.091	0.074	0.060	0.049	0.040	0.033	0.027	0.022	0.018	0.015	0.012	0.010	0.009	0.007	0.006	0.002	0.001	0.000
24	0.788	0.622	0.492	0.390	0.310	0.247	0.197	0.158	0.126	0.102	0.082	0.066	0.053	0.043	0.035	0.028	0.023	0.019	0.015	0.013	0.010	0.008	0.007	0.006	0.005	0.002	0.001	0.000
25	0.780	0.610	0.478	0.375	0.295	0.233	0.184	0.146	0.116	0.092	0.074	0.059	0.047	0.038	0.030	0.024	0.020	0.016	0.013	0.010	0.009	0.007	0.006	0.005	0.004	0.001	0.001	0.000
26	0.772	0.598	0.464	0.361	0.281	0.220	0.172	0.135	0.106	0.084	0.066	0.053	0.042	0.033	0.026	0.021	0.017	0.014	0.011	0.009	0.007	0.006	0.005	0.004	0.003	0.001	0.000	0.000
27	0.764	0.586	0.450	0.347	0.268	0.207	0.161	0.125	0.098	0.076	0.060	0.047	0.037	0.029	0.023	0.018	0.014	0.011	0.009	0.007	0.006	0.005	0.004	0.003	0.002	0.001	0.000	0.000
28	0.757	0.574	0.437	0.333	0.255	0.196	0.150	0.116	0.090	0.069	0.054	0.042	0.033	0.026	0.020	0.016	0.012	0.010	0.008	0.006	0.005	0.004	0.003	0.002	0.002	0.001	0.000	0.000
29	0.749	0.563	0.424	0.321	0.243	0.185	0.141	0.107	0.082	0.063	0.048	0.037	0.029	0.022	0.017	0.014	0.011	0.008	0.006	0.005	0.004	0.003	0.002	0.002	0.002	0.000	0.000	0.000
30	0.742	0.552	0.412	0.308	0.231	0.174	0.131	0.099	0.075	0.057	0.044	0.033	0.026	0.020	0.015	0.012	0.009	0.007	0.005	0.004	0.003	0.003	0.002	0.002	0.001	0.000	0.000	0.000

Table 2: Present value of an annuity of 1 unit received at the end of each of the next n periods assuming a constant discount rate of r per period

n \ r	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	11%	12%	13%	14%	15%	16%	17%	18%	19%	20%	21%	22%	23%	24%	25%	30%	35%	40%
1	0.990	0.980	0.971	0.962	0.952	0.943	0.935	0.926	0.917	0.909	0.901	0.893	0.885	0.877	0.870	0.862	0.855	0.847	0.840	0.833	0.826	0.820	0.813	0.806	0.800	0.769	0.741	0.714
2	1.970	1.942	1.913	1.886	1.859	1.833	1.808	1.783	1.759	1.736	1.713	1.690	1.668	1.647	1.626	1.605	1.585	1.566	1.547	1.528	1.509	1.492	1.474	1.457	1.440	1.361	1.289	1.224
3	2.941	2.884	2.829	2.775	2.723	2.673	2.624	2.577	2.531	2.487	2.444	2.402	2.361	2.322	2.283	2.246	2.210	2.174	2.140	2.106	2.074	2.042	2.011	1.981	1.952	1.816	1.696	1.589
4	3.902	3.808	3.717	3.630	3.546	3.465	3.387	3.312	3.240	3.170	3.102	3.037	2.974	2.914	2.855	2.798	2.743	2.690	2.639	2.589	2.540	2.494	2.448	2.404	2.362	2.166	1.997	1.849
5	4.853	4.713	4.580	4.452	4.329	4.212	4.100	3.993	3.890	3.791	3.696	3.605	3.517	3.433	3.352	3.274	3.199	3.127	3.058	2.991	2.926	2.864	2.803	2.745	2.689	2.436	2.220	2.035
6	5.795	5.601	5.417	5.242	5.076	4.917	4.767	4.623	4.486	4.355	4.231	4.111	3.998	3.889	3.784	3.685	3.589	3.498	3.410	3.326	3.245	3.167	3.092	3.020	2.951	2.643	2.385	2.168
7	6.728	6.472	6.230	6.002	5.786	5.582	5.389	5.206	5.033	4.868	4.712	4.564	4.423	4.288	4.160	4.039	3.922	3.812	3.706	3.605	3.508	3.416	3.327	3.242	3.161	2.802	2.508	2.263
8	7.652	7.325	7.020	6.733	6.463	6.210	5.971	5.747	5.535	5.335	5.146	4.968	4.799	4.639	4.487	4.344	4.207	4.078	3.954	3.837	3.726	3.619	3.518	3.421	3.329	2.925	2.598	2.331
9	8.566	8.162	7.786	7.435	7.108	6.802	6.515	6.247	5.995	5.759	5.537	5.328	5.132	4.946	4.772	4.607	4.451	4.303	4.163	4.031	3.905	3.786	3.673	3.566	3.463	3.019	2.665	2.379
10	9.471	8.983	8.530	8.111	7.722	7.360	7.024	6.710	6.418	6.145	5.889	5.650	5.426	5.216	5.019	4.833	4.659	4.494	4.339	4.192	4.054	3.923	3.799	3.682	3.571	3.092	2.715	2.414
11	10.368	9.787	9.253	8.760	8.306	7.887	7.499	7.139	6.805	6.495	6.207	5.938	5.687	5.453	5.234	5.029	4.836	4.656	4.486	4.327	4.177	4.035	3.902	3.776	3.656	3.147	2.752	2.438
12	11.255	10.575	9.954	9.385	8.863	8.384	7.943	7.536	7.161	6.814	6.492	6.194	5.918	5.660	5.421	5.197	4.988	4.793	4.611	4.439	4.278	4.127	3.985	3.851	3.725	3.190	2.779	2.456
13	12.134	11.348	10.635	9.986	9.394	8.853	8.358	7.904	7.487	7.103	6.750	6.424	6.122	5.842	5.583	5.342	5.118	4.910	4.715	4.533	4.362	4.203	4.053	3.912	3.780	3.223	2.799	2.469
14	13.004	12.106	11.296	10.563	9.899	9.295	8.745	8.244	7.786	7.367	6.982	6.628	6.302	6.002	5.724	5.468	5.229	5.008	4.802	4.611	4.432	4.265	4.108	3.962	3.824	3.249	2.814	2.478
15	13.865	12.849	11.938	11.118	10.380	9.712	9.108	8.559	8.061	7.606	7.191	6.811	6.462	6.142	5.847	5.575	5.324	5.092	4.876	4.675	4.489	4.315	4.153	4.001	3.859	3.268	2.825	2.484
16	14.718	13.578	12.561	11.652	10.838	10.106	9.447	8.851	8.313	7.824	7.379	6.974	6.604	6.265	5.954	5.668	5.405	5.162	4.938	4.730	4.536	4.357	4.189	4.033	3.887	3.283	2.834	2.489
17	15.562	14.292	13.166	12.166	11.274	10.477	9.763	9.122	8.544	8.022	7.549	7.120	6.729	6.373	6.047	5.749	5.475	5.222	4.990	4.775	4.576	4.391	4.219	4.059	3.910	3.295	2.840	2.492
18	16.398	14.992	13.754	12.659	11.690	10.828	10.059	9.372	8.756	8.201	7.702	7.250	6.840	6.467	6.128	5.818	5.534	5.273	5.033	4.812	4.608	4.419	4.243	4.080	3.928	3.304	2.844	2.494
19	17.226	15.678	14.324	13.134	12.085	11.158	10.336	9.604	8.950	8.365	7.839	7.366	6.938	6.550	6.198	5.877	5.584	5.316	5.070	4.843	4.635	4.442	4.263	4.097	3.942	3.311	2.848	2.496
20	18.046	16.351	14.877	13.590	12.462	11.470	10.594	9.818	9.129	8.514	7.963	7.469	7.025	6.623	6.259	5.929	5.628	5.353	5.101	4.870	4.657	4.460	4.279	4.110	3.954	3.316	2.850	2.497
21	18.857	17.011	15.415	14.029	12.821	11.764	10.836	10.017	9.292	8.649	8.075	7.562	7.102	6.687	6.312	5.973	5.665	5.384	5.127	4.891	4.675	4.476	4.292	4.121	3.963	3.320	2.852	2.498
22	19.660	17.658	15.937	14.451	13.163	12.042	11.061	10.201	9.442	8.772	8.176	7.645	7.170	6.743	6.359	6.011	5.696	5.410	5.149	4.909	4.690	4.488	4.302	4.130	3.970	3.323	2.853	2.498
23	20.456	18.292	16.444	14.857	13.489	12.303	11.272	10.371	9.580	8.883	8.266	7.718	7.230	6.792	6.399	6.044	5.723	5.432	5.167	4.925	4.703	4.499	4.311	4.137	3.976	3.325	2.854	2.499
24	21.243	18.914	16.936	15.247	13.799	12.550	11.469	10.529	9.707	8.985	8.348	7.784	7.283	6.835	6.434	6.073	5.746	5.451	5.182	4.937	4.713	4.507	4.318	4.143	3.981	3.327	2.855	2.499
25	22.023	19.523	17.413	15.622	14.094	12.783	11.654	10.675	9.823	9.077	8.422	7.843	7.330	6.873	6.464	6.097	5.766	5.467	5.195	4.948	4.721	4.514	4.323	4.147	3.985	3.329	2.856	2.499
26	22.795	20.121	17.877	15.983	14.375	13.003	11.826	10.810	9.929	9.161	8.488	7.896	7.372	6.906	6.491	6.118	5.783	5.480	5.206	4.956	4.728	4.520	4.328	4.151	3.988	3.330	2.856	2.500
27	23.560	20.707	18.327	16.330	14.643	13.211	11.987	10.935	10.027	9.237	8.548	7.943	7.409	6.935	6.514	6.136	5.798	5.492	5.215	4.964	4.734	4.524	4.332	4.154	3.990	3.331	2.856	2.500
28	24.316	21.281	18.764	16.663	14.898	13.406	12.137	11.051	10.116	9.307	8.602	7.984	7.441	6.961	6.534	6.152	5.810	5.502	5.223	4.970	4.739	4.528	4.335	4.157	3.992	3.331	2.857	2.500
29	25.066	21.844	19.188	16.984	15.141	13.591	12.278	11.158	10.198	9.370	8.650	8.022	7.470	6.983	6.551	6.166	5.820	5.510	5.229	4.975	4.743	4.531	4.337	4.159	3.994	3.332	2.857	2.500
30	25.808	22.396	19.600	17.292	15.372	13.765	12.409	11.258	10.274	9.427	8.694	8.055	7.496	7.003	6.566	6.177	5.829	5.517	5.235	4.979	4.746	4.534	4.339	4.160	3.995	3.332	2.857	2.500

Table 3: Cumulative Normal Distribution Table

d	$N(d)$	d	$N(d)$	d	$N(d)$	d	$N(d)$	d	$N(d)$	d	$N(d)$
−3.00	0.0013	−1.58	0.0571	−0.76	0.2236	0.06	0.5239	0.86	0.8051	1.66	0.9515
−2.95	0.0016	−1.56	0.0594	−0.74	0.2297	0.08	0.5319	0.88	0.8106	1.68	0.9535
−2.90	0.0019	−1.54	0.0618	−0.72	0.2358	0.10	0.5398	0.90	0.8159	1.70	0.9554
−2.85	0.0022	−1.52	0.0643	−0.70	0.2420	0.12	0.5478	0.92	0.8212	1.72	0.9573
−2.80	0.0026	−1.50	0.0668	−0.68	0.2483	0.14	0.5557	0.94	0.8264	1.74	0.9591
−2.75	0.0030	−1.48	0.0694	−0.66	0.2546	0.16	0.5636	0.96	0.8315	1.76	0.9608
−2.70	0.0035	−1.46	0.0721	−0.64	0.2611	0.18	0.5714	0.98	0.8365	1.78	0.9625
−2.65	0.0040	−1.44	0.0749	−0.62	0.2676	0.20	0.5793	1.00	0.8414	1.80	0.9641
−2.60	0.0047	−1.42	0.0778	−0.60	0.2743	0.22	0.5871	1.02	0.8461	1.82	0.9656
−2.55	0.0054	−1.40	0.0808	−0.58	0.2810	0.24	0.5948	1.04	0.8508	1.84	0.9671
−2.50	0.0062	−1.38	0.0838	−0.56	0.2877	0.26	0.6026	1.06	0.8554	1.86	0.9686
−2.45	0.0071	−1.36	0.0869	−0.54	0.2946	0.28	0.6103	1.08	0.8599	1.88	0.9699
−2.40	0.0082	−1.34	0.0901	−0.52	0.3015	0.30	0.6179	1.10	0.8643	1.90	0.9713
−2.35	0.0094	−1.32	0.0934	−0.50	0.3085	0.32	0.6255	1.12	0.8686	1.92	0.9726
−2.30	0.0107	−1.30	0.0968	−0.48	0.3156	0.34	0.6331	1.14	0.8729	1.94	0.9738
−2.25	0.0122	−1.28	0.1003	−0.46	0.3228	0.36	0.6406	1.16	0.8770	1.96	0.9750
−2.20	0.0139	−1.26	0.1038	−0.44	0.3300	0.38	0.6480	1.18	0.8810	1.98	0.9761
−2.15	0.0158	−1.24	0.1075	−0.42	0.3373	0.40	0.6554	1.20	0.8849	2.00	0.9772
−2.10	0.0179	−1.22	0.1112	−0.40	0.3446	0.42	0.6628	1.22	0.8888	2.05	0.9798
−2.05	0.0202	−1.20	0.1151	−0.38	0.3520	0.44	0.6700	1.24	0.8925	2.10	0.9821
−2.00	0.0228	−1.18	0.1190	−0.36	0.3594	0.46	0.6773	1.26	0.8962	2.15	0.9842
−1.98	0.0239	−1.16	0.1230	−0.34	0.3669	0.48	0.6844	1.28	0.8997	2.20	0.9861
−1.96	0.0250	−1.14	0.1271	−0.32	0.3745	0.50	0.6915	1.30	0.9032	2.25	0.9878
−1.94	0.0262	−1.12	0.1314	−0.30	0.3821	0.52	0.6985	1.32	0.9066	2.30	0.9893
−1.92	0.0274	−1.10	0.1357	−0.28	0.3897	0.54	0.7054	1.34	0.9099	2.35	0.9906
−1.90	0.0287	−1.08	0.1401	−0.26	0.3974	0.56	0.7123	1.36	0.9131	2.40	0.9918
−1.88	0.0301	−1.06	0.1446	−0.24	0.4052	0.58	0.7191	1.38	0.9162	2.45	0.9929
−1.86	0.0314	−1.04	0.1492	−0.22	0.4129	0.60	0.7258	1.40	0.9192	2.50	0.9938
−1.84	0.0329	−1.02	0.1539	−0.20	0.4207	0.62	0.7324	1.42	0.9222	2.55	0.9946
−1.82	0.0344	−1.00	0.1587	−0.18	0.4286	0.64	0.7389	1.44	0.9251	2.60	0.9953
−1.80	0.0359	−0.98	0.1635	−0.16	0.4365	0.66	0.7454	1.46	0.9279	2.65	0.9960
−1.78	0.0375	−0.96	0.1685	−0.14	0.4443	0.68	0.7518	1.48	0.9306	2.70	0.9965
−1.76	0.0392	−0.94	0.1736	−0.12	0.4523	0.70	0.7580	1.50	0.9332	2.75	0.9970
−1.74	0.0409	−0.92	0.1788	−0.10	0.4602	0.72	0.7642	1.52	0.9357	2.80	0.9974
−1.72	0.0427	−0.90	0.1841	−0.08	0.4681	0.74	0.7704	1.54	0.9382	2.85	0.9978
−1.70	0.0446	−0.88	0.1894	−0.06	0.4761	0.76	0.7764	1.56	0.9406	2.90	0.9981
−1.68	0.0465	−0.86	0.1949	−0.04	0.4841	0.78	0.7823	1.58	0.9429	2.95	0.9984
−1.66	0.0485	−0.84	0.2005	−0.02	0.4920	0.80	0.7882	1.60	0.9452	3.00	0.9986
−1.64	0.0505	−0.82	0.2061	0.00	0.5000	0.82	0.7939	1.62	0.9474	3.05	0.9989
−1.62	0.0526	−0.80	0.2119	0.02	0.5080	0.84	0.7996	1.64	0.9495		
−1.60	0.0548	−0.78	0.2177	0.04	0.5160						