PRIFYSGOL ABERYSTWYTH UNIVERSITY

ADRAN MATHEMATEG / DEPARTMENT OF MATHEMATICS

ARHOLIADAU SEMESTER 1 / SEMESTER 1 EXAMINATIONS

IONAWR / JANUARY 2026

MA10110 – Coordinate and Vector Geometry

The questions on this paper are written in English.

Amser a ganiateir - 2 awr

- Rhoddir marciau llawn am atebion cyflawn i bob cwestiwn yn Rhan A ac i dri cwestiwn yn Rhan B.
- Yn Rhan B, rhoddir credyd am y tri ateb gorau.
- Ni cheir defnyddio cyfrifianellau.
- Mae modd i fyfyrwyr gyflwyno atebion i'r papur hwn naill ai yn y Gymraeg neu'r Saesneg.

Time allowed - 2 hours

- Full marks will be given for complete answers to all questions in Section A and to three questions in Section B.
- In Section B, credit will be given for the best three answers.
- Calculators are not permitted.
- Students may submit answers to this paper in either Welsh or English.

The following formulae may be assumed:

- The perpendicular distance of a point P:(x', y') from a line $ax + by + c = 0$ is
$$\frac{ax' + by' + c}{\sqrt{a^2 + b^2}}.$$
- Two lines with slopes m_1 and m_2 meet at an angle $\theta = \tan^{-1}\left(\frac{m_2 - m_1}{1 + m_1 m_2}\right)$.
- The equation of the tangent to the circle $x^2 + y^2 + 2ax + 2by + c = 0$ at the point P:(x_1, y_1) is
$$(x_1 + a)x + (y_1 + b)y + ax_1 + by_1 + c = 0.$$

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Section A

1. Give, in general form, the equations of the lines through the point $(1, 4)$ that are parallel and perpendicular to the line $4x - 3y + 1 = 0$. [5 marks]

2. Determine, using the form $F(x, y) + \lambda G(x, y) = 0$, the equation of the line through the intersection of the lines $4x - 3y + 1 = 0$ and $2x + y - 8 = 0$ with slope 3, giving your answer in general form. [5 marks]

3. Determine which of the points $(1, 3)$, $(2, 2)$ and $(-5, -6)$ are on the same side of the line $4x - 3y + 1 = 0$ as the origin. Determine the inequality that c must satisfy for the point $(5, c)$ to be on the same side of this line as the origin. [5 marks]

4. A conic has focus at $(0, 0)$ and directrix at $x = -2$. The point $(1, 3)$ lies on the conic. Hence determine the eccentricity of the conic, and state whether it is an ellipse, parabola or hyperbola. Determine if the point $(2, 4)$ could lie on the same conic. [5 marks]

5. Find the cosine of the angle between the vectors $\underline{a} = \underline{i} - \underline{j} + 2\underline{k}$ and $\underline{b} = 3\underline{i} - \underline{j} + 3\underline{k}$. Hence find the projection of $\underline{b}$ on $\underline{a}$. [5 marks]

6. Find the vector equation of the line formed by the intersection of the planes $A : 4x + 3y + z = 1$, and $B : 2x + 4y - z = -2$. [5 marks]

7. Determine a unit vector normal to the plane $6x + 2y - 3z = 1$. [5 marks]

8. $OABC$ is a trapezium with $\overrightarrow{OA} = \underline{a}$, $\overrightarrow{OC} = \underline{c}$ and $\overrightarrow{AB}$ parallel to, and half the length of, $\overrightarrow{OC}$. The point D divides BC in the ratio $1 : 3$. Sketch the trapezium and find the vector $\overrightarrow{OD}$ in terms of $\underline{a}$ and $\underline{c}$. [5 marks]

Section B

9. (a) Find the shortest distance of the point $(1, 4)$ from the line $4x - 3y + 5 = 0$. [3 marks]

(b) Determine the tangent of the acute angle (i.e. $\tan\theta$) between the lines $4x - 3y + 5 = 0$ and $x + y = 0$. [4 marks]

(c) Find the angular bisectors of the lines $4x - 3y + 5 = 0$ and $y = 0$. [4 marks]

(d) Give the equation of the tangent to the circle $x^2 + y^2 - 24x - 2y + 45 = 0$ at the point $(4, 7)$. [6 marks]

(e) Show that the point $(1, 4)$ is outside this circle and find its tangential distance. [3 marks]

10. (a) Describe, without proofs, the locus of the point P in the following cases, where A, B and C are fixed distinct points:

(i) $\widehat{APB}$ is a right angle. [2 marks]

(ii) $\widehat{APB} = \widehat{ACB}$. [3 marks]

(b) A is the point $(3, 1)$, and the point K moves on the circle $(x-1)^2 + y^2 = 1$. Find the locus of the midpoint of KA. [7 marks]

(c) A point P moves such that its distance from the point $(1, 3)$ is twice its distance from the point $(-2, 0)$. Determine an equation for the locus of P and describe the conic section that it represents. [8 marks]

11. (a) Find the Cartesian equation of the curve $r^2 \sin(2\theta) = 4$. [3 marks]

(b) Determine the polar equation for the circle $x^2 + (y + 2)^2 = 4$. [2 marks]

(c) In parametric form, $P : (2at, at^2)$ and $Q : (2as, as^2)$ are two points on the parabola $x^2 = 4ay$.

(i) Explain briefly why $(2at, at^2)$ is a suitable parametrisation for this parabola. [1 mark]

(ii) If the line joining P and Q has slope equal to one, give a relationship between s and t. [3 marks]

(iii) Determine the (x, y) coordinates of the point of intersection of the normals to the parabola at P and Q. [8 marks]

(iv) Use the relationship from part (ii) to show that as t and s vary the intersection point from part (iii) moves in a straight line. [3 marks]

12. (a) Given points A and B with position vectors $\underline{a} = 2\underline{i} + 3\underline{j}$ and $\underline{b} = -\underline{i} - 4\underline{j}$, determine:

(i) the vector equation of the line through A orthogonal to $\underline{b}$; [2 marks]

(ii) the vector equation of the line through the origin orthogonal to $\underline{a}$. [2 marks]

(b) Find the distance of the point A, with position vector given in (a), from the line $\underline{r} = \underline{i} - 4\underline{j} + \lambda\left(\underline{i} - \underline{j}\right)$. [6 marks]

(c) It is Freshers Fair, and everyone is waiting for the free pizza to arrive. The position vectors of students A and B are as given in (a). Student A can run at speed $5\mathrm{m/s}$ and student B at speed $9\mathrm{m/s}$. The pizza P is brought to the door with position vector $\overrightarrow{OP} = 8\underline{i} + 11\underline{j}$ m. Assuming that the position vectors given above for A and B are in metres, and that as soon as the pizza arrives the students run towards it, find:

(i) The direction $\overrightarrow{AP}$ in which student A moves. [1 mark]

(ii) The velocity vector $\underline{v}_B$ describing the motion of B. [2 marks]

(iii) The velocity of B relative to A. [2 marks]

(iv) Which of A or B reaches the pizza first. [5 marks]

13. (a) Find the Cartesian equation of the plane passing through $(1, 2, -1)$, normal to $\underline{i} - \underline{j} + 5\underline{k}$. [3 marks]

(b) Evaluate the vector product of $\underline{i} - \underline{j} + 5\underline{k}$ and $2\underline{i} - \underline{j} + 2\underline{k}$. [3 marks]

(c) Find a unit vector perpendicular to the plane containing the points $P : (1, -1, 3)$, $Q : (2, -2, 8)$, $R : (3, -2, 5)$. [5 marks]

(d) Hence, or otherwise, give the equation of the plane. [3 marks]

(e) Find the perpendicular distance of the point $S : (1, 2, -1)$ from this plane. [3 marks]

(f) Find the area of the triangle with vertices P, Q and R. [3 marks]

End of exam paper