PRIFYSGOL ABERYSTWYTH UNIVERSITY

ADRAN MATHEMATEG / DEPARTMENT OF MATHEMATICS

ARHOLIADAU SEMESTER 1 / SEMESTER 1 EXAMINATIONS

IONAWR / JANUARY 2026

MP10610 – Calculus

The questions on this paper are written in English.

Amser a ganiateir - 2 awr

- Rhoddir marciau llawn am atebion cyflawn i bob cwestiwn yn Rhan A ac i dri cwestiwn yn Rhan B.
- Yn Rhan B, rhoddir credyd am y tri ateb gorau.
- Cyfrifianellau Casio FX-83 neu FX-85 YN UNIG a ganiateir.
- Mae modd i fyfyrwyr gyflwyno atebion i'r papur hwn naill ai yn y Gymraeg neu'r Saesneg.

Time allowed - 2 hours

- Full marks will be given for complete answers to all questions in Section A and to three questions in Section B.
- In Section B, credit will be given for the best three answers.
- Casio FX-83 or FX-85 calculators ONLY may be used.
- Students may submit answers to this paper in either Welsh or English.

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Section A

1. Show that the function

$$f(x) = \begin{cases} 4 - x^2 & \text{if } x \leq 0, \\ 4 + x & \text{if } x > 0, \end{cases}$$

is continuous but not differentiable at $x = 0$. [5 marks]

2. Determine the limits

(a)

$$\lim_{x \to 1} \frac{x^2 - 3x + 2}{1 - x},$$

(b)

$$\lim_{x \to \infty} \frac{1 - \sqrt{x}}{1 + \sqrt{x}}.$$

[2,3 marks]

3. Differentate the functions

(a)

$$f(x) = e^x \sin x,$$

(b)

$$g(x) = \frac{x^2 + 1}{e^x}.$$

[2,3 marks]

4. Find the first and second derivative of

$$f(x) = \sqrt{\sin x}.$$

[5 marks]

5. Find the function f if $f''(x) = x^{-3/2}$, $f'(4) = 7$ and $f(0) = 0$. [5 marks]

6. Use the substitution $u = x^2 + 1$ to evaluate

$$\int_0^1 x\sqrt{x^2+1}\,\mathrm{d}x.$$

[5 marks]

7. Use integration by parts to evaluate the integral

$$\int_1^8 \frac{2\ln x}{x^2}\,\mathrm{d}x.$$

[5 marks]

8. Find the absolute minimum and maximum values of $g(x) = -x^3+3x^2-6,\ -2 \leq x \leq 3$.

[5 marks]

Section B

9. (a) Suppose that f and g are differentiable functions. Show *from first principles* that

$$\frac{d}{dx}\left(f(x)g(x)\right) = f'(x)g(x) + f(x)g'(x).$$

[5 marks]

(b) Differentiate $f(x) = kx^3$ from first principles (where k is a constant). [5 marks]

(c) Find dy/dx if

$$(3x - y)(2x + 3y) = 8.$$

[5 marks]

(d) Use logarithmic differentiation to find dy/dx if

$$y = (2x - 5)^{3x}.$$

[5 marks]

10. (a) Consider the hyperbolic sine function

$$\sinh x = \frac{1}{2}\left(e^x - e^{-x}\right).$$

Determine whether the function is odd, even or neither. [2 marks]

(b) Show that

$$\sinh^{-1} x = \ln\left(x + \sqrt{x^2 + 1}\right).$$

[7 marks]

(c) Show that

$$\frac{d}{dx}\left(\sinh^{-1} x\right) = \frac{1}{\sqrt{x^2 + 1}}.$$

[5 marks]

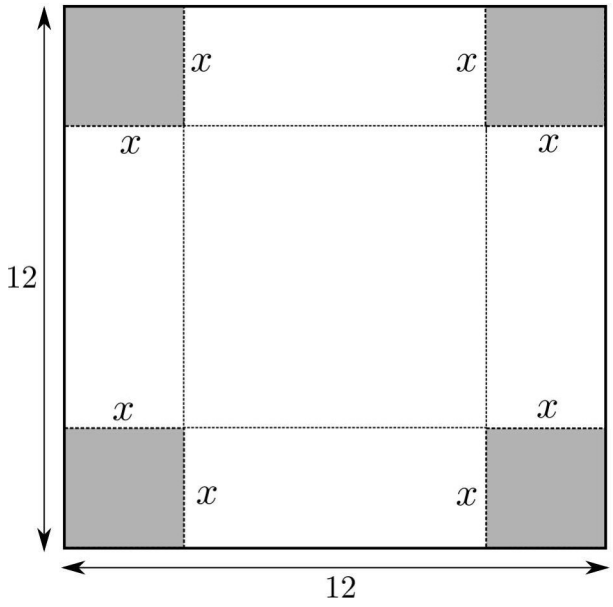
(d) Find

$$\int \sinh^{-1} x \, \mathrm{d}x.$$

[6 marks]

. . . questions continue

11. (a) A square piece of cardboard, 12 cm by 12 cm, is to be made into an open box. Squares with sides x cm are to be cut from each of the four corners and each side of the box will then be folded up.



Find the maximum volume that the box can hold. [10 marks]

(b) The acceleration function (in meters per second squared) for a particle moving in a straight line is $a(t) = 2(t-1),\ 1 \leq t \leq 4$.

(i) Find the velocity v at time t if $v(1) = -4$.

(ii) Find the displacement of the particle over the given time interval.

(iii) Find the distance traveled by the particle over this time.

[3,3,4 marks]

12. Consider the function

$$f(x) = \frac{x^2 - 4}{x^2 - 4x}.$$

(a) Determine the domain and range of f. [3 marks]

(b) Find the intercepts of $y = f(x)$ with the x and y axes. [2 marks]

(c) Determine whether f is an odd function, even function or neither. [2 marks]

(d) Find the horizontal, vertical and/or slant asymptotes to $y = f(x)$. [3 marks]

(e) Determine and characterise any critical points of f, and state the intervals of increase and decrease. [5 marks]

(f) Sketch the curve $y = f(x)$. [5 marks]

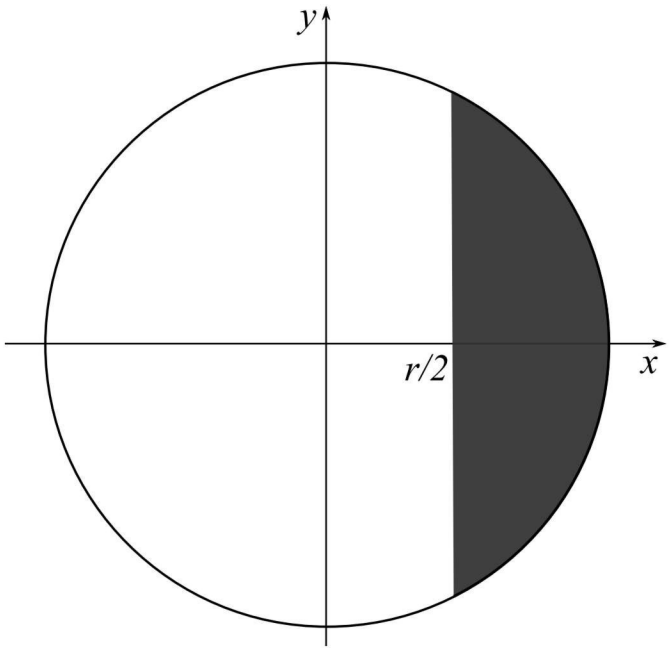
. . . questions continue

13. (a) Evaluate

$$\int \frac{\cos x}{\sin^2 x - \sin x}\, dx$$

using an appropriate substitution. [8 marks]

(b) In the figure below, the shaded portion is the region inside the circle $x^2 + y^2 = r^2$ for $r/2 \leq x \leq r$. Use integration to find the area of the shaded region. [12 marks]



End of exam paper