PRIFYSGOL ABERYSTWYTH UNIVERSITY

ADRAN MATHEMATEG / DEPARTMENT OF MATHEMATICS

ARHOLIADAU SEMESTER 2 / SEMESTER 2 EXAMINATIONS

MAI – MEHEFIN / MAY – JUNE 2026

MA01020 – Mathematics Tutorial

The questions on this paper are written in English.

Amser a ganiateir - 2 awr

- Rhoddir marciau llawn am atebion cyflawn i bob cwestiwn yn Rhan A ac i dri cwestiwn yn Rhan B.
- Yn Rhan B, rhoddir credyd am y tri ateb gorau.
- Ni cheir defnyddio cyfrifianellau.
- Mae taflen fformiwlâu wedi cael ei hatodi.
- Mae modd i fyfyrwyr gyflwyno atebion i'r papur hwn naill ai yn y Gymraeg neu'r Saesneg.

Time allowed - 2 hours

- Full marks will be given for complete answers to all questions in Section A and to three questions in Section B.
- In Section B, credit will be given for the best three answers.
- Calculators are not permitted.
- A formula sheet is attached.
- Students may submit answers to this paper in either Welsh or English.

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Section A

1. (a) Find x if $x^2 - 9 = x + 3,\ x > 0$.

(b) Simplify $\log_2 72 - 2\log_2 3$ to single number. [5 marks]

2. (a) Find all values of n where $2 < 2n + 3 \leq 9, n \in \mathbb{Z}$.

(b) Solve $8 \leq 32 - 4x < 26, x \in \mathbb{R}$, giving your answer as a Real number interval. [5 marks]

3. By completing the square, solve $2x^2 - 12x - 7 = 0$. [5 marks]

4. An arithmetic sequence has first term $a_1 = -11$ and series sum $S_{10} = 25$.

(a) Find the common difference, d.

(b) Find the 25$^{\text{th}}$ term, a_{25}. [5 marks]

5. If $f(x) = 2x^3 + 10x^2 - 5kx - 2k$ has a factor $x + 2$, use the Factor Theorem to find k. [5 marks]

6. Simplify each of the following complex number expressions to a single complex number in Cartesian form:

(a) $(4 - \mathrm{i})(3 + 2\mathrm{i})$;

(b) $\dfrac{2 - \mathrm{i}}{1 + 3\mathrm{i}}$. [5 marks]

7. Define the vectors: $\mathbf{u} = \mathbf{i} - 5\mathbf{j} + 2\mathbf{k},\ \mathbf{v} = -2\mathbf{i} + 4\mathbf{k},\ \mathbf{w} = 2\mathbf{j} - 3\mathbf{k}$. Find:

(a) the scalar product $(\mathbf{u} - \mathbf{v}) \cdot \mathbf{w}$;

(b) and the vector product $\mathbf{v} \times \mathbf{w}$. [5 marks]

8. Consider the statements:
A: "The sum of any 3 consecutive odd numbers is a multiple of 3."
B: "The sum of any 3 odd numbers is a multiple of 3."

State if each is true or false. Prove any true statement and give a counter-example to disprove any false statement. [5 marks]

Section B

9. (a) Consider the following sets:

$$A := \{n \in \mathbb{N} : n = 3k, k \in \mathbb{N}, n < 40\};$$
$$B := \{n \in \mathbb{N} : n = k(k+1)/2, k \in \mathbb{N}, k \leq 6\};$$
$$C := \{x \in \mathbb{R} : 6 < x \leq 21\}.$$

Find: (i) $A \cap B$; (ii) $(A \cup B) \cap C$. [5 marks]

(b) Solve $x^2 + 3x > 10$, giving your answer using set (or Real number interval) notation. [4 marks]

(c) Let $f(x) = 2x^3 + 9x^2 + 7x - 6$.

(i) Show that $x + 2$ is a factor of $f(x)$. [1 mark]

(ii) Use algebraic long division to fully factorise $f(x)$. [6 marks]

(d) Evaluate $\sin(\pi/12)$, expressing your answer in surd form. [4 marks]

10. (a) Solve $3^{x^2-4} = 27^{2-x}$. [5 marks]

(b) (i) Write the complex number $z = -3\sqrt{3} + 3\mathrm{i}$ in polar form. [4 marks]

(ii) Hence, find all unique solutions to $w^4 = -3\sqrt{3} + 3\mathrm{i},\ w \in \mathbb{C}$. [6 marks]

(c) Consider the following functions:

$$f : \mathbb{R}^+ \to \mathbb{R}^+; f(x) = \sqrt{x};$$
$$g : \mathbb{R}^+ \to \mathbb{R}; g(x) = \log x,$$

where $\mathbb{R}^+ = \{x \in \mathbb{R} : x > 0\}$.
Which of the compositions $f \circ g$ and $g \circ f$ are possible? Explain your answer and give the full definition for the valid composition(s). [5 marks]

11. (a) (i) Simplify $\left(\sqrt{a} + \sqrt{b}\right)^2 + \left(\sqrt{a} - \sqrt{b}\right)^2, \quad a > 0, b > 0.$ [3 marks]

(ii) Hence, or otherwise, evaluate $\left(\sqrt{5} + \sqrt{3}\right)^2 + \left(\sqrt{5} - \sqrt{3}\right)^2$. [2 marks]

(b) Consider the line $y - x = 5$, the circle $(x+2)^2 + (y+1)^2 = 10$, and the points $\mathrm{A}{:}(-1, 3), \mathrm{B}{:}(-5, -1), \mathrm{C}{:}(-4, 2)$. Determine algebraically:

(i) which of the points are on the same side of the line; [3 marks]

(ii) which of the points are inside the circle. [3 marks]

(iii) where intersection points of the line and circle are located; [5 marks]

(iv) the equation of the tangent line to the circle at $(-3, 2)$. [4 marks]

12. (a) If the equation $3x^2 - 5x + 1 = 0$ has roots p, q, using Vieta's Theorem, find a quadratic equation with roots $3p, 3q$. [6 marks]

(b) Find all solutions to $2\sin^2\theta - \cos\theta = 1,\ 0 \leq \theta < 2\pi$. [6 marks]

(c) Prove by mathematical induction that for all $n \in \mathbb{N}$,

$$1 + 3 + 5 + \cdots + (2n - 1) = n^2.$$

[8 marks]

13. (a) (i) Sketch the graph of $y = f(x) = x^2 - 2x - 6$, clearly labelling the x and y intercepts, the turning point, and the axis of symmetry of the graph.[5 marks]

(ii) On the same axes, shade the area enclosed by $y = f(x)$ and the line $y = 2$, clearly labelling the intersection points between the line and the curve. [3 marks]

(b) Give the complex number $4\,\mathrm{e}^{5\pi\mathrm{i}/4}$ in Cartesian form. [3 marks]

(c) Let $f : [1, 5] \to [0, 16] : f(x) = x^2 - 2x + 1$.
Find the inverse of f, giving the full definition (including its domain and codomain). [5 marks]

(d) Using De Moivre's Theorem, show that $\cos 2\theta = \cos^2\theta - \sin^2\theta$. [4 marks]

Useful Formulae and Theorems

Set theory: A set with n elements has 2^n subsets, including $\emptyset$.

Arithmetic sequence with terms $a_1, a_2, \ldots, a_n$, common difference d and series n^{th} term S_n:

$$a_n = a_1 + (n-1)d$$
$$S_n = \frac{n}{2}\left[2a_1 + (n-1)d\right]$$

Geometric sequence with terms $u_1, u_2, \ldots, u_n$, common ratio r and series n^{th} term S_n:

$$u_n = u_1 r^{n-1}$$
$$S_n = \frac{u_1(1-r^n)}{1-r}, \quad r \neq 1$$
$$S_\infty = \frac{u_1}{1-r}, \quad |r| < 1$$

Trigonometry:

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha$$
$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\beta\sin\alpha$$

Given two vectors $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ and $\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$, where $\mathbf{i}, \mathbf{j}, \mathbf{k}$ are unit vectors in the x, y, z directions respectively,

- the scalar product is given by $\mathbf{a} \cdot \mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3$;
- the vector product is given by $\mathbf{a} \times \mathbf{b} = (a_2b_3 - a_3b_2)\mathbf{i} - (a_1b_3 - a_3b_1)\mathbf{j} + (a_1b_2 - a_2b_1)\mathbf{k}$.

Vieta's Theorem states that if a quadratic equation $ax^2 + bx + c = 0$ has roots p, q, then

$$p + q = \frac{-b}{a};$$
$$pq = \frac{c}{a}.$$

De Moivre's Theorem states that for $z \in \mathbb{C}$,

$$z^n = [r(\cos\theta + \mathrm{i}\sin\theta)]^n = r^n(\cos n\theta + \mathrm{i}\sin n\theta), \quad n \in \mathbb{N}.$$