PRIFYSGOL ABERYSTWYTH UNIVERSITY

**ADRAN MATHEMATEG / DEPARTMENT OF MATHEMATICS**

**ARHOLIADAU SEMESTER 2 / SEMESTER 2 EXAMINATIONS**

**MAI – MEHEFIN / MAY – JUNE 2026**

# MA03610 – Foundations of Mathematics II

The questions on this paper are written in English.

**Amser a ganiateir - 2 awr**

- Rhoddir marciau llawn am atebion cyflawn i bob cwestiwn yn Rhan A ac i dri cwestiwn yn Rhan B.
- Yn Rhan B, rhoddir credyd am y tri ateb gorau.
- Cyfrifianellau Casio FX-83 neu FX-85 YN UNIG a ganiateir.
- Mae modd i fyfyrwyr gyflwyno atebion i'r papur hwn naill ai yn y Gymraeg neu'r Saesneg.

**Time allowed - 2 hours**

- Full marks will be given for complete answers to all questions in Section A and to three questions in Section B.
- In Section B, credit will be given for the best three answers.
- Casio FX-83 or FX-85 calculators ONLY may be used.
- Students may submit answers to this paper in either Welsh or English.

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## Section A

**1**. Given the matrices

$$A = \begin{bmatrix} 2 & -1 \\ -3 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 4 \\ 0 & -2 \end{bmatrix},$$

find $2A - 3B$, $B^T$, and $BA$. [5 marks]

**2**. Write the simultaneous equations

$$\begin{cases} 2x + 5y &= 0, \\ x + 2y &= 1, \end{cases}$$

in the form of a matrix equation. Solve for $x$ and $y$ by inverting the matrix of coefficients. [5 marks]

**3**. Find the $2 \times 2$ transformation matrix that represents

(a) a reflection in the $x$-axis (the line $y = 0$),

(b) a rotation counter-clockwise by $90°$ ($\pi/2$ radians),

(c) a reflection in the $x$-axis followed by a rotation by $90°$.

[5 marks]

**4**. Integrate the following function with respect to $x$:

$$4x^3 - \frac{3}{x} + 2\sin(4x) + e^{-x}.$$

[5 marks]

**5**. Evaluate the definite integral

$$\int_1^2 \left(3 + \frac{2 - x^3}{x^2}\right) \mathrm{d}x.$$

[5 marks]

**6**. Evaluate the definite integral

$$\int_0^{\sqrt{\pi/2}} x \cos\left(x^2\right) \mathrm{d}x.$$

[5 marks]

**7**. Find the particular solution of the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = e^{2x} - 3x^2, \quad y(0) = 1.$$

[5 marks]

**8**. Find the explicit general solution of the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{y + 1}{3x^3},$$

(i.e. express $y$ explicitly as a function of $x$). [5 marks]

## Section B

**9**. (a) Rearrange the following four matrix equations to make $X$ the subject:

$$\text{(i) } 4X - 2B = A, \qquad \text{(ii) } AX = 2X + AC,$$
$$\text{(iii) } XB = XC^T - B, \qquad \text{(iv) } AXC = A^T AC.$$

[6 marks]

(b) Suppose we have

$$A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 3 & 5 \\ 2 & 7 \\ 5 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 5 \\ 0 & 2 & 0 \end{bmatrix}.$$

In each of the cases (i)-(iv) above, determine whether the matrix $X$ is well-defined, justifying your answer. If $X$ is well defined, compute it. [14 marks]

**10**. (a) Consider the simultaneous equations

$$\begin{cases} x + 2y + 3z &= 2, \\ -x + 4y + z &= 0, \\ x - 3y - z &= 3. \end{cases}$$

(i) Express the equations in the form of a matrix equation. Demonstrate that there is a unique solution for $x, y, z$. [4 marks]

(ii) Use row operations to find the solution $x, y, z$, indicating clearly which operations you use. [9 marks]

(b) The rectangle $R$ has vertices at the points $(1, 1)$, $(1, a)$, $(3, 1)$, and $(3, a)$, where $a > 0$ is a positive constant. The rectangle is transformed onto the shape $R' = AR$ by the transformation matrix

$$A = \begin{bmatrix} 1 & 1 \\ -a & 0 \end{bmatrix}.$$

Given that the area of rectangle $R'$ is $4$ square units, find the value of $a$. [7 marks]

**11**. Integrate the following functions with respect to $x$

$$\text{(a) } \frac{4x}{(x+1)(x-1)^2}, \quad \text{(b) } x^2 e^{3x}, \quad \text{(c) } x \ln\left(x^2\right).$$

[8,5,7 marks]

**12**. (a) A new piece of equipment is being added to a children's play area. One side will be covered with decorated plywood, whose surface area needs to be determined. The side is bounded from below by the $x$-axis between $0$ and $10$, and from above by the graph of the function

$$f(x) = \begin{cases} \sin\left(\dfrac{\pi x}{2}\right), & 0 \leq x < 1, \\ 1, & 1 \leq x \leq 3, \\ 1 - (x-3)(x-6), & 3 < x \leq 6, \\ \exp\left(6 - x\right), & 6 < x \leq 10, \end{cases}$$

where $\exp(.)$ denotes the exponential function. Compute the surface area of the side of the play equipment. [12 marks]

(b) The arc length of a function $y(x)$ is given by

$$S = \int \sqrt{1 + \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}\, \mathrm{d}x.$$

If $y(x) = \dfrac{1}{8}x^2 - \ln(x)$, compute the arc length over $x \in [1, 2]$. You may use without proof that

$$\frac{1}{16}x^2 + \frac{1}{2} + x^{-2} = \left(\frac{1}{4}x + x^{-1}\right)^2.$$

[8 marks]

**13**. A small object falling under gravity has its velocity $v(t)$ given by the equation

$$m\frac{\mathrm{d}v}{\mathrm{d}t} = mg - cv(t), \tag{1}$$

where $m$ is its mass, $c$ the wavespeed of the medium, and $g$ the gravitational constant.

(a) The object reaches its terminal velocity when $\dfrac{\mathrm{d}v}{\mathrm{d}t} = 0$. Using equation (1), show that the object's terminal velocity is $v = mg/c$. [2 marks]

(b) By solving the differential equation (1), show that the velocity of the object over time is given by

$$v(t) = \frac{mg}{c} + B\exp\left(-\frac{c}{m}t\right), \tag{2}$$

for some constant $B$, where $\exp(.)$ denotes the exponential function. [8 marks]

(c) If the object is initially stationary, find the value of the constant $B$. [2 marks]

(d) The position of the object $x(t)$ is related to the velocity as $v(t) = \dfrac{\mathrm{d}x}{\mathrm{d}t}$. Using this relation and equation (2), determine an explicit expression for the position of the object as a function of time. [8 marks]