PRIFYSGOL ABERYSTWYTH UNIVERSITY

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DEPARTMENT OF PHYSICS

SEMESTER 1 EXAMINATIONS, JANUARY 2026

PH06020 – Introduction to Mathematical Methods 1

Time allowed – 3 hours

- Full marks can be attained for complete answers to ALL eight questions in Section A and TWO questions from Section B
- The numbers in brackets after the questions indicate the allocation of marks for each question.
- **No calculators** are allowed to be used.
- A booklet of mathematical formulae and standard constants will be provided.
- MAE MODD I FYFYRWYR GYFLWYNO ATEBION I'R PAPUR HWN NAILL AI YN Y GYMRAEG NEU'R SAESNEG.
- STUDENTS MAY SUBMIT ANSWERS TO THIS PAPER IN EITHER WELSH OR ENGLISH.

Once seated, students may complete the front cover of the answer book(s) and the attendance slip ONLY.

DO NOT TURN THE EXAMINATION QUESTION PAPER OVER UNTIL INSTRUCTED TO DO SO.

List of Formulae

Fractions

$$\frac{a}{b}+\frac{c}{d}\equiv\frac{ad+bc}{bd}$$

$$\frac{\left(\frac{a}{b}\right)}{\left(\frac{c}{d}\right)}\equiv\frac{a}{b}\times\frac{c}{d}\equiv\frac{ac}{bd}$$

Trigonometry

$$180°=\pi \qquad 1\text{ rad}=\frac{180°}{\pi}$$

$$\sin^2 x+\cos^2 x\equiv 1$$

$$\tan x=\frac{\sin x}{\cos x} \quad \cot x=\frac{\cos x}{\sin x}$$

$$\operatorname{cosec} x=\frac{1}{\sin x} \quad \sec x=\frac{1}{\cos x}$$

Exponential Functions

(with $b>0$ and $d>0$)

$$b^0\equiv 1 \qquad b^1\equiv b \qquad b^a\equiv b^c\leftrightarrow a=c$$

$$b^a b^c\equiv b^{a+c} \qquad \frac{b^a}{b^c}\equiv b^{a-c}$$

$$(b^a)^c\equiv(b^c)^a\equiv b^{ac} \quad (bd)^a\equiv b^a d^a$$

Logarithmic Functions

(with $a,b,c>0$ and $b\neq 1$)

$$\log_b a=d\leftrightarrow b^d=a$$

$$\log_b(b^d)\equiv d \qquad b^{\log_b a}\equiv a$$

$$\log_b a=\log_b c\leftrightarrow b=a$$

$$\log_b 1\equiv 0 \qquad \log_b b\equiv 1$$

$$\log_b(a^d)\equiv d\log_b a$$

$$\log_b(ac)\equiv\log_b a+\log_b c$$

$$\log_b\left(\frac{a}{c}\right)\equiv\log_b a-\log_b c$$

$$\log_b a\equiv\frac{\log_c a}{\log_c b}$$

Complex Numbers

(with real a, b, r, and α)

$$(a+ib)^*\equiv a-ib \qquad \left(re^{i\alpha}\right)^*\equiv re^{-i\alpha}$$

$$z^*z\equiv r^2 \quad a=r\cos\alpha \quad b=r\sin\alpha$$

$$r=\sqrt{a^2+b^2} \qquad \cos\alpha=\frac{a}{r} \qquad \sin\alpha=\frac{b}{r}$$

Vectors

$$\underline{\mathbf{a}}.\underline{\mathbf{b}}\equiv|\underline{\mathbf{a}}|.|\underline{\mathbf{b}}|\cos\gamma$$

$$(a_1,a_2,a_3).(b_1,b_2,b_3)\equiv a_1b_1+a_2b_2+a_3b_3$$

$$\underline{\mathbf{a}}\times\underline{\mathbf{b}}\equiv\left(|\underline{\mathbf{a}}|.|\underline{\mathbf{b}}|\sin\gamma\right)\underline{\mathbf{n}}$$

Where $|\mathbf{n}|=1$ and $\underline{\mathbf{n}}\perp\underline{\mathbf{a}},\underline{\mathbf{b}}$

$$\underline{\mathbf{a}}\times\underline{\mathbf{b}}\equiv-(\underline{\mathbf{b}}\times\underline{\mathbf{a}})$$

$$\underline{\mathbf{i}}\times\underline{\mathbf{i}}=\underline{\mathbf{j}}\times\underline{\mathbf{j}}=\underline{\mathbf{k}}\times\underline{\mathbf{k}}=0$$

$$\underline{\mathbf{a}}\times\underline{\mathbf{b}}=-(\underline{\mathbf{b}}\times\underline{\mathbf{a}})$$

$$\underline{\mathbf{i}}\times\underline{\mathbf{j}}=\underline{\mathbf{k}},\ \underline{\mathbf{j}}\times\underline{\mathbf{k}}=\underline{\mathbf{i}},\ \underline{\mathbf{k}}\times\underline{\mathbf{i}}=\underline{\mathbf{j}}$$

$$(a_1,a_2,a_3)\times(b_1,b_2,b_3)$$

$$=(a_2b_3-a_3b_2,a_3b_1-a_1b_3,a_1b_2-a_2b_1)$$

Differentiation

$$\frac{d}{dx}(x^a)=ax^{a-1}$$

$$\frac{d}{dx}(\sin x)=\cos x \quad \frac{d}{dx}(\cos x)=-\sin x$$

$$\frac{d}{dx}(\exp x)=\exp x \quad \frac{d}{dx}(\ln x)=\frac{1}{x}$$

$$\frac{d}{dx}(Af_1+Bf_2)\equiv A\frac{df_1}{dx}+B\frac{df_2}{dx}$$

Section A

Attempt ALL the eight questions.

1 Evaluate: [5]

$$F = \frac{\frac{2}{5} + \frac{1}{6}}{\frac{3}{5} - \frac{5}{6}}$$

and round it to two decimal places (2dp).

2 Solve the equations simultaneously and check the solutions. [5]

$$x + y = z$$
$$5x + 6z + 3 = y$$
$$5x = 21 + y + 2z$$

3 Make a sketch of a unit circle, show roughly the point on it that represents $\alpha = 456°$, and determine the signs of $\sin\alpha$, $\cos\alpha$, $\tan\alpha$, $\cot\alpha$, $\sec\alpha$, and $\operatorname{cosec}\alpha$. [5]

4 Evaluate: [5]

$$T = \log_6 21 + \log_6 10 - \log_6 7 + \log_6 2 - \log_6 5 + \log_6 3$$

5 Find $\sin x$ where: [5]

$11(1 + \sin x)\sin x = 5(3 - \cos^2 x)$

6 Find vector $\mathbf{c} = \mathbf{a} \times \mathbf{b}$, where $\mathbf{a} = 9\mathbf{i} + 8\mathbf{j} + 5\mathbf{k}$ and $\mathbf{b} = 10\mathbf{i} + 6\mathbf{j} + 7\mathbf{k}$ and derive the magnitude of the vector product, $c = |\mathbf{c}|$. [5]

7 Give the two square roots of $z = 50\sqrt{2} - i50\sqrt{2}$ in exponential form. [5]

8 Determine the gradient of $y = 4x^3 - 7x^2 + 3x - 8$ at $x = 3$. [5]

Section B

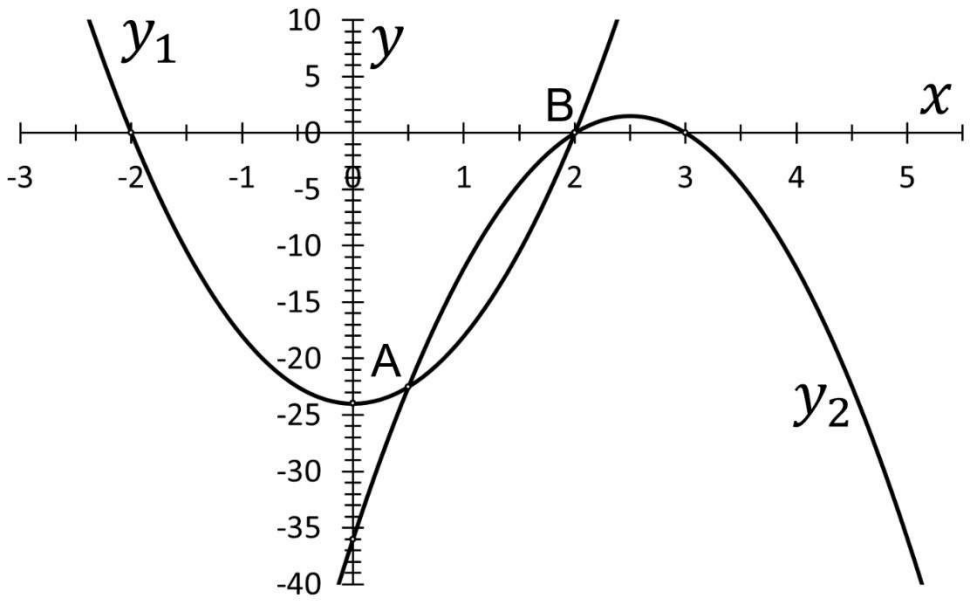
Answer TWO of the three questions.

9 a) Evaluate:

$$C = \frac{-5 + i14}{-4 + i}$$

[5]

b) The plot below shows two parabolas.



i) Give the standard form of the two quadratic functions, y_1 and y_2.

[6]

ii) Work out the (x, y) coordinates of points A and B, the two intercepts of the parabolas.

[9]

10 a) Evaluate:

$$H = \frac{8^3 \times 4^{-6} \times 16^3}{(2^5)^2}$$

[5]

b) Vectors $\mathbf{a}$ and $\mathbf{b}$ are:

$\mathbf{a} = 2\mathbf{i} - 5\mathbf{j} + 4\mathbf{k}$ and $\mathbf{b} = -5\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}$

i) Using the scalar product of $\mathbf{a}$ and $\mathbf{b}$, find the cosine function of the angle between the two vectors.

[5]

ii) Using the vector product of $\mathbf{a}$ and $\mathbf{b}$, find the sine function of the angle between the two vectors.

[5]

iii) Show that both vectors $\mathbf{a}$ and $\mathbf{b}$ are perpendicular to their vector product.

[4]

iv) Using the unit circle or otherwise, give an estimate value of the angle between vectors $\mathbf{a}$ and $\mathbf{b}$.

[1]

11 a) Evaluate:

$$S = \tan 1050°$$

[6]

b) The linear function of **Line 1** is: $y_1 = -x + 4$

Line 2 is obtained by stretching Line 1 vertically (from the x axis, along the y axis) by factor 2.

Line 3 is obtained by shifting Line 2 to the left (along the x axis in the negative direction) by 2.

Line 4 is obtained by reflecting Line 3 vertically (to the x axis).

Line 5 is obtained by stretching Line 4 horizontally (from the y axis, along the x axis) by factor 2.

Determine the (x, y) coordinates of the point where line 1 and line 5 intersect.

[14]

END OF PAPER