PRIFYSGOL ABERYSTWYTH UNIVERSITY

ADRAN FFISEG

DEPARTMENT OF PHYSICS

SEMESTER 1 EXAMINATIONS, JANUARY 2026

PH16210 – Algebra and Differential Equations

Time allowed – 2 hours

- **Full marks can be attained for complete answers to ALL six questions in Section A and TWO questions from Section B**
- **The numbers in brackets after the questions indicate the allocation of marks for each question.**
- **Only calculators from the Casio FX83, FX85 or FX991 range may be used.**
- **A booklet of mathematical formulae and standard constants will be provided.**
- **MAE MODD I FYFYRWYR GYFLWYNO ATEBION I'R PAPUR HWN NAILL AI YN Y GYMRAEG NEU'R SAESNEG**
- **STUDENTS MAY SUBMIT ANSWERS TO THIS PAPER IN EITHER WELSH OR ENGLISH**

Once seated, students may complete the front cover of the answer book(s) and the attendance slip ONLY.

DO NOT TURN THE EXAMINATION QUESTION PAPER OVER UNTIL INSTRUCTED TO DO SO.

Section A

Attempt ALL questions in the booklets provided

1 A quadratic polynomial is given by [5]

$$p(x) = a\,x^2 + 5x + 7.$$

Determine the values of the coefficent a such that the roots of the quadratic are real. In particular, give the values of a for which there is only one root.

2 Determine the values of the real and imaginary parts x and y in the complex fraction below: [5]

$$\frac{2+i}{1+5i} = x + iy.$$

3 Compute the Taylor expansion to second order of the function [5]

$$f(x) = e^x \cos x.$$

4 Find the polynomials $q(x)$ and $r(x)$ such that [5]

$$x^4 + x^3 + 2x^2 - x + 3 = (x^3 - x - 2)\,q(r) + r(x)$$

where $r(x)$ has degree less than 3.

5 For what real values of x does the complex number $z = x + i\frac{1}{\sqrt{2}}$ lie on the unit circle? For the values x you calculate, give the polar form of these complex numbers z. [5]

6 The velocity v of a falling particle is modelled by the differential equation [5]

$$\frac{dv}{dt} = -mg - kv - cv^2 + f\,(1 - e^{-at})$$

where m, g, k, c, f, a are constants. In this equation,

i) Identify the dependent variable;

ii) Identify the independent variable;

iii) Give the order of the differential equation;

iv) State whether it is linear or nonlinear

v) State whether it is homogeneous or inhomogeneous.

Section B

Answer TWO questions

7 Consider the family of complex numbers of the form

$$z(\theta) = e^{i\theta} + 1$$

with θ a real angle.

a) Determine the real and imaginary parts of $z(\theta) = x(\theta) + i\, y(\theta)$ in terms of the angle θ. From this, show that the modulus is

$$|z(\theta)| = 2\left|\cos\frac{\theta}{2}\right|.$$

[5]

b) For which values of θ is $|z(\theta)|^2$ a maximum and a minimum. Compute the values of the modulus-squared for these values.

[5]

c) Find all the roots of the polynomial $z^6 - z^4 = 0$. Sketch their position in the complex plane and comment on any degenerate roots.

[5]

8 a) An object is falling under gravity and experiences air resistance. Its equation of motion is

$$m\frac{dv}{dt} = mg - m\gamma\, v.$$

Here $\gamma > 0$ is the damping term.

Show that the equation may be rewritten as $\frac{dv}{dt} = -\gamma\,(v - v_\infty)$ where v_∞ is a constant to be determined.

Give the physical interpretation of v_∞ and comment on the physical interpretation of the differential equation for the cases $v < v_\infty$ and $v > v_\infty$.

[4]

b) A first-order differential equation has solutions of the form $y = C\,x^3 + 1$ where C is an arbitrary constant. Determine the differential equation.

[4]

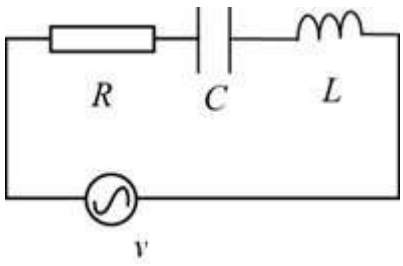
c) Solve the differential equation for $x > 0$

$$\frac{dy}{dx} = \frac{x}{y} + 2.$$

Given that $xln(x) \to 0$ as $x \to 0$ from above, comment on any unusual features of this equation with regards to setting initial conditions at $x = 0$.

[7]

9 An electrical circuit can be represented as a capacitor, an inductor and a resistor connected in series across a voltage source $v(t)$ The values of the components are $R = 2\ \Omega$, and $L = 1\ \mathrm{H}$. The capacitance C is tunable.



a) Write down the second-order differential equation for the charge $q(t)$.

[2]

b) For what values of C is the circuit underdamped?

[3]

c) Determine the general solution to the homogeneous equation $(v = 0)$ in the critically damped case.

[3]

d) In the critically damped case, determine the general solution equation when the voltage is given by

$$v(t) = 3\cos(5t).$$

[7]

END OF PAPER