PRIFYSGOL ABERYSTWYTH UNIVERSITY

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DEPARTMENT OF PHYSICS

SEMESTER 1 EXAMINATIONS, JANUARY 2026

PH33310 – Engineering Control Theory

Time allowed – 2 hours

- **Full marks can be attained for complete answers to ALL six questions in Section A and TWO questions from Section B**
- **The numbers in brackets after the questions indicate the allocation of marks for each question.**
- **Only calculators from the Casio FX83, FX85 or FX991 range may be used.**
- **A booklet of mathematical formulae and standard constants will be provided.**
- **Only a 1 side of A4 size Summary Sheet is allowed.**
- **MAE MODD I FYFYRWYR GYFLWYNO ATEBION I'R PAPUR HWN NAILL AI YN Y GYMRAEG NEU'R SAESNEG**
- **STUDENTS MAY SUBMIT ANSWERS TO THIS PAPER IN EITHER WELSH OR ENGLISH**

Once seated, students may complete the front cover of the answer book(s) and the attendance slip ONLY.

DO NOT TURN THE EXAMINATION QUESTION PAPER OVER UNTIL INSTRUCTED TO DO SO.

Section A

Attempt ALL questions in the booklets provided

1 State the defining properties of a Linear Time-Invariant system. [5]

2 A linear time-invariant system has the transfer function [5]

$$G(s) = \frac{(s+1)(s-5)}{s(s^2+s+2)}.$$

Find the zeroes and poles of G. What is the type of the system. Comment on the stability of the system.

3 State the Final Value Theorem. Using the fact that $\mathcal{L}(e^{at}) = \frac{1}{s-a}$ compute [5]

the Laplace transform of $cos(\omega t)$. Does this meet the conditions for the

Final Value Theorem to hold?

4 The step function $u(t)$ takes the value 1 for $t \geq 0$ and the value 0 for $t <$ [5]

0. Determine its Laplace transform. A transfer function takes the form $G(s) = \frac{1}{s^r}H(s)$ where $H(0)$ is a fixed constant. Deduce the long-time asymptotic value of the output when the input is the step function.

5 A state-based model for a MIMO system is given by [5]

$$\dot{x} = Ax + Bu,$$

$$y = Cx + Du.$$

Here x is a vector of state variables and A, B, C, D are matrices. Describe the decomposition into controllable and uncontrollable state variables.

6 Using Laplace transform methods, write the transfer function matrix $G(s) = \frac{Y(s)}{U(s)}$ for the state-based model appearing in question 5. Give a sufficient condition for stability. [5]

Section B

Answer TWO questions

7 A circuit consists of a capacitor and inductor in series. The energy stored in the capaitor is $\frac{1}{2C}\ q^2$ while the energy stored in the inductor is $\frac{1}{2}L\ \dot{q}^2$ where q is the charge and $i = \dot{q}$ is the current. The Lagrangian for the circuit is

$$L(q, \dot{q}, t) = \frac{1}{2}L\ \dot{q}^2 - \frac{1}{2C}\ q^2 + v(t)q.$$

a) From Lagrangian's equation $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = 0$, derive the circuit equation.

[4]

b) Determine the conjugate momentum variable $p = \frac{\partial L}{\partial \dot{q}}$ and give the Hamiltonian.

[3]

c) A low pass filter is given below.



The input and output voltages are related by $RC\ \frac{dv_{out}}{dt} + v_{out} = v_{in}$.

Determine the transfer function $G(s) = \frac{V_{out}(s)}{V_{in}(s)}$ for the filter.

[3]

d) State how the transfer function determines the gain and the phase shift for the output when the input is a harmonic signal $v_{in}(t) \propto \cos(\omega t)$.

[2]

e) Draw the Bode plots for the low pass filter.

[8]

8 A plant takes the form of an undamped harmonic oscillator with transfer function given by

$$G(s) = \frac{s}{s^2+1}.$$

a) Sketch the root locus for this transfer function.

[7]

b) The plant is placed in loop as shown below.

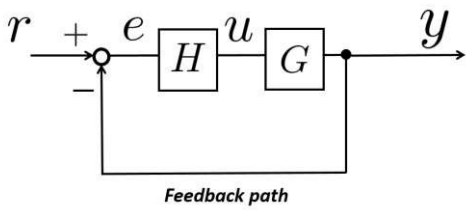


Figure 1

Show that the closed loop transfer function is given by

$$G_{fb}(s) = \frac{sH(s)}{1+s^2+sH(s)}.$$

[1]

d) If we take the controller to be a PID filter with transfer function

$$H(s) = s + k + \frac{a}{s},$$

Describe the root locus as we vary $k > 0$ with a fixed. How does this compare to the simpler proportional controller?

[7]

9 A linear time-invariant SISO system has the state space representation

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = [\,1 \quad 0 \quad 0\,]\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + 0\,u.$$

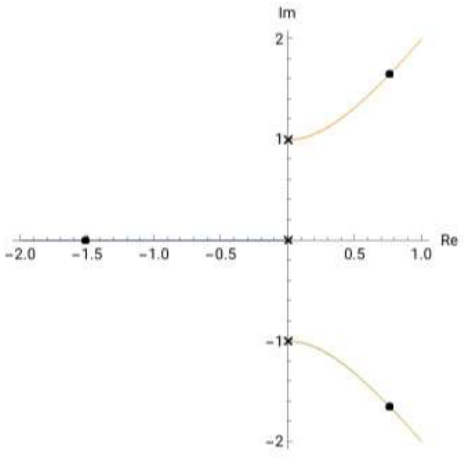
a) Show that the transfer function is given by $G(s) = \frac{1}{s(1+s^2)}$.

[5]

b) Are there any uncontrollable or unobservable state variables in this representation?

[3]

c) Determine the zeroes and poles of G. The root locus is shown below. Explain the main features and say whether the system can be stabilizabled for any gain, whether positive or negative.



[7]

END OF PAPER