PRIFYSGOL ABERYSTWYTH UNIVERSITY

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DEPARTMENT OF PHYSICS

SEMESTER 1 EXAMINATIONS, JANUARY 2026

PHM2510 – Electromagnetic Theory

Time allowed – 2 hours

- **Full marks can be attained for complete answers to ALL six questions in Section A and TWO questions from Section B**
- **The numbers in brackets after the questions indicate the allocation of marks for each question.**
- **Only calculators from the Casio FX83, FX85 or FX991 range may be used.**
- **A booklet of mathematical formulae and standard constants will be provided.**
- **Only a 1 side of A4 summary sheet is allowed.**
- **MAE MODD I FYFYRWYR GYFLWYNO ATEBION I'R PAPUR HWN NAILL AI YN Y GYMRAEG NEU'R SAESNEG**
- **STUDENTS MAY SUBMIT ANSWERS TO THIS PAPER IN EITHER WELSH OR ENGLISH**

Once seated, students may complete the front cover of the answer book(s) and the attendance slip ONLY.

DO NOT TURN THE EXAMINATION QUESTION PAPER OVER UNTIL INSTRUCTED TO DO SO.

Section A

Attempt ALL questions in the booklets provided

1 The Poynting vector is given by

$$\mathbf{N} = \mathbf{E} \times \mathbf{H}$$

In the usual notation, explain the meaning of Poynting vector and its dimension and the other two quantities in the above equation.

[2]

The power of the sunlight (often known as solar constant) at the orbit of Earth is 1361 W/m². Assuming that the light can be treated as plane waves, estimate the mean magnitude of sunlight's electric field.

[3]

2 When an electromagnetic wave with angular frequency ω is propagating in a plasma, the relative permittivity of the plasma for the wave is given by

$$\varepsilon = 1 - \frac{\omega_p^2}{\omega^2},$$

where ω_p is the angular plasma frequency. The phase speed of the wave is $\frac{\omega}{k} = \frac{c}{\sqrt{\varepsilon}}$. The wave's group velocity (which is the gradient of the angular wave frequency in the wavenumber space or $\frac{d\omega}{dk}$) represents the speed that energy is transported by the wave. With the expression of the phase speed, show that the group velocity is always smaller than the speed of light in free space. Here *c* is the speed of light in free space, *k* is the wavenumber.

[5]

3 A student experiments with a light which is a plane electromagnetic wave. From vacuum he lets the wave strike on the surface of a dielectric material with a relative permittivity $\varepsilon = 2.3$. He finds that at a specific angle of incidence, the reflected light disappears completely. Find the angle of incidence when the

reflected light disappears and state the name given to this angle in optics. What relationship of the electric field and the magnetic field of the wave to the plane of incidence can be found from this experiment?

[5]

4 The vector potential $\mathbf{A}$ and the scalar potential ϕ are not uniquely specified in the solutions to Maxwell's equations. Using the solution to Gauss's Law for magnetism describe non-uniqueness. [5]

5 The azimuthal electric field component for a Hertzian dipole is given by:

$$E_\theta = \frac{-1}{4\pi\epsilon_0} k^2 p_0 e^{j(\omega t - kr)} \left[1 - \frac{j}{kr} - \frac{1}{k^2 r^2}\right] \frac{1}{r} \sin(\theta)$$

Explain what all of the terms are in the equation. How could this equation be simplified when the far field approximation holds?

[5]

6 A stationary observer measures the length of a bullet train to be 350 m, moving at a speed of 0.5c. Consider relativistic effects and calculate the actual length of the train when at rest.

(Take the speed of light, c = 3.0×10^8 ms^{-1}).

[5]

Section B

Answer TWO questions

7 A rectangular waveguide has internal dimensions a = 4.8 cm (width) and b = 2.4 cm (height), and is air-filled (both the relative permittivity and relative permeability are equal to 1). The waveguide operates in the dominant TE_{10} mode at a frequency of f =10 GHz.

a) Calculate the cut-off frequency (f_c) of the TE_{10} mode. Show your working and express the answer in GHz.

[3]

b) Show that the waveguide wavelength $\lambda_g = \frac{2\pi}{k_g}$ can be expressed as

$$\lambda_g = \frac{\lambda_0}{\sqrt{1-(f_c/f)^2}}.$$

Determine the guide wavelength at 10 GHz. Express your answer in m.

[6]

c) Calculate the phase speed and group velocity of the TE_{10} mode at 10 GHz. Express both in terms of speed of light in free space and also give numerical values in m/s.

[6]

8 a) Clearly explain what *a retarded potential* is, and why it must be accounted for? Why is it often difficult to measure?

[6]

b) Define a half-wave antenna. Draw the schematic of a halfwave dipole antenna and its radiated power plot.

[4]

c) Starting from the retarded potential, explain in words the steps we would take in order to derive analytical expressions for an antenna's power.

[5]

9 a) Define a four-vector in the context of special relativity. State the components of the position four-vector and explain what they refer to.

[3]

b) Describe the electric and magnetic fields produced by a stationary charge and explain how these fields appear to an observer moving with a constant velocity relative to the charge.

[4]

c) An astronaut is travelling through space at a constant velocity of 0.8c. Observers on Earth measure the duration of the journey to be 5 years.

i) Using the concept of time dilation, calculate the time experienced by the astronaut on board the spacecraft.

ii) If the spacecraft instead travels at 0.99c, calculate how much time the astronaut would experience for the same journey.

iii) Compare the results of (i) and (ii) and discuss how the change in astronaut speed impacts the time experienced in the spacecraft. (Take the speed of light, $c = 3.0 \times 10^8\ ms^{-1}$).

[8]

END OF PAPER