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DEPARTMENT OF PHYSICS

SEMESTER 2 EXAMINATIONS, MAY 2026

PH06520 – Introduction to Mathematical Methods 2

Time allowed – 3 hours

- Full marks can be attained for complete answers to ALL eight questions in Section A and TWO questions from Section B
- The numbers in brackets after the questions indicate the allocation of marks for each question.

- **Calculators** are **not** allowed to be used.
- A booklet of mathematical formulae and standard constants will be provided.

- Mae modd i fyfyrwyr gyflwyno atebion i'r papur hwn naill ai yn y Gymraeg neu'r Saesneg.
- Students may submit answers to this paper in either Welsh or English.

Once seated, students may complete the front cover of the answer book(s) and the attendance slip ONLY.

Do NOT turn the examination question paper over until instructed to do so.

List of Formulas

$\sqrt{x} \equiv x^{\frac{1}{2}}$ $\sqrt{x^3} \equiv x^{\frac{3}{2}}$ $\frac{1}{\sqrt{x}} \equiv x^{-\frac{1}{2}}$ $e \approx 2.7(18\ldots)$ $\pi \approx 3.14(159\ldots)$

Differentiation

f:	C	x^k	$\sin x$	$\cos x$	e^x	$\ln x$
$\frac{df}{dx}$:	0	kx^{k-1}	$\cos x$	$-\sin x$	e^x	$\frac{1}{x}$

$\frac{d}{dx}(Af + Bg) = A\frac{df}{dx} + B\frac{dg}{dx}$ $\frac{d}{dx}(fg) = \frac{df}{dx}g + f\frac{dg}{dx}$ $\frac{d}{dx}\left(\frac{f}{g}\right) = \frac{\frac{df}{dx}g - f\frac{dg}{dx}}{g^2}$

$\frac{d}{dx}\Big(f\big(g(x)\big)\Big) = \frac{df}{du}\frac{du}{dx},\ u \equiv g(x)$

$\frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)}$ $L(x,y) = R(x,y) \rightarrow \frac{dL}{dx} = \frac{dR}{dx} \rightarrow \frac{dy}{dx} = f(x,y)$

Sequences and Series

$a_n = a_k + (n-k)d$ $S_n\left(\equiv \sum_{j=1}^{n} a_j\right) = n(n-1)\frac{a_1 + a_n}{2} = n\left(a_1 + \frac{n-1}{2}d\right)$

$a_n = a_k r^{n-k}$ $S_n\left(\equiv \sum_{j=1}^{n} a_j\right) = \frac{1-r^n}{1-r}a_1$ $S\left(\equiv \sum_{j=1}^{\infty} a_j\right) = \frac{a_1}{1-r}$ $-1 < r < 1$

$(a+b)^n = \sum_{k=0}^{n} C_k^n a^{n-k}b^k,$ $C_k^n \equiv \frac{n!}{k!\,(n-k)!},$ $s! \equiv s \times (s-1) \times (s-2) \times \ldots \times 3 \times 2$

Matrices

$B = sA \leftrightarrow B_{ij} = sA_{ij}$ $C = A \pm B \leftrightarrow C_{ij} = A_{ij} \pm B_{ij}$ $C = AB \leftrightarrow C_{ij} = \sum_{k=1}^{n} A_{ik}B_{kj}$

$b = Ma \leftrightarrow b_i = \sum_{k=1}^{n} M_{ik}a_k$ $b = aM \leftrightarrow b_i = \sum_{k=1}^{n} a_k M_{ki}$

$\det\begin{bmatrix} a & b \\ c & d \end{bmatrix} \equiv \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

$MM^{-1} = M^{-1}M = I$ $\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

Integration

$$\frac{dF}{dx} = f \leftrightarrow \int f dx = F + C \qquad \int_{x_1}^{x_2} f dx = [F]_{x_1}^{x_2} \equiv F(x_2) - F(x_1)$$

$$\int (Af + Bg)dx = A\int f dx + B\int g dx$$

f:	A	$x^k, \quad k \neq -1$	$\sin x$	$\cos x$	e^x	$\frac{1}{x}$
$\int f dx$:	$Ax + C$	$\frac{x^{k+1}}{k+1} + C$	$-\cos x + C$	$\sin x + C$	$e^x + C$	$\ln\lvert x\rvert + C$

$$\int_{x_1}^{x_2} u\frac{dv}{dx}dx = [uv]_{x_1}^{x_2} - \int_{x_1}^{x_2} \frac{du}{dx}v dx$$

$$A = \int_{x_1}^{x_2} (y_{\text{top}} - y_{\text{bottom}})dx \qquad V = \pi\int_{x_1}^{x_2} y^2 dx \qquad L = \int_{x_1}^{x_2} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$\frac{ax + b}{(x - c)(x - d)} = \frac{A}{x - c} + \frac{B}{x - d}$$

Trigonometry

$\sin^2 x + \cos^2 x \equiv 1$

$\sin(a + b) \equiv \sin a \cos b + \cos a \sin b$

$\sin(a - b) \equiv \sin a \cos b - \cos a \sin b$

$\cos(a + b) \equiv \cos a \cos b - \sin a \sin b$

$\cos(a - b) \equiv \cos a \cos b + \sin a \sin b$

$\sin(2x) \equiv 2 \sin x \cos x$

$\cos(2x) \equiv \cos^2 x - \sin^2 x \equiv 2\cos^2 x - 1 \equiv 1 - 2\sin^2 x$

$$\sec x \equiv \frac{1}{\cos x} \qquad \operatorname{cosec} x \equiv \frac{1}{\sin x} \qquad \tan x \equiv \frac{\sin x}{\cos x} \qquad \cot x \equiv \frac{1}{\tan x} = \frac{\cos x}{\sin x}$$

Section A

Attempt ALL questions in the booklet provided.

1 The positive integers ending with 3 form an arithmetic sequence: [5]

$a_1, a_2, a_3, \ldots = 3, 13, 23, \ldots$

a) Determine the 50th element, a_{50}, of the sequence.

b) Calculate the sum of the first 100 positive integers ending with 3:

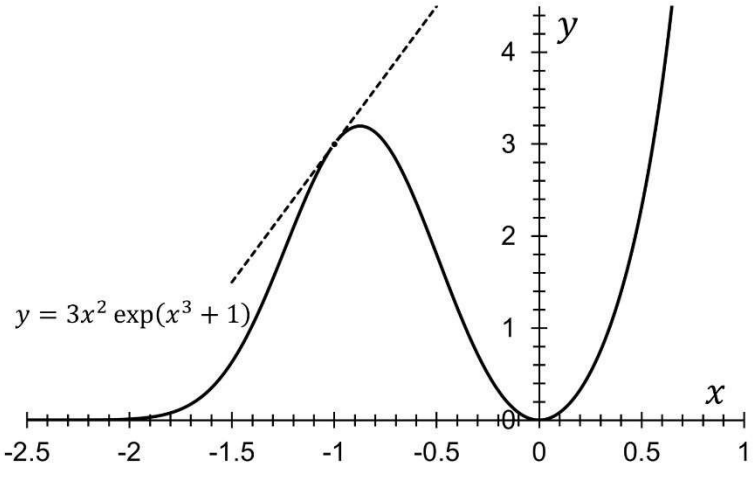
$S_{100} = a_1 + a_2 + \cdots + a_{100}$

2 a) With respect to x, differentiate:

$y = 3x^2 \exp(x^3 + 1)$

[3]

b) And evaluate the derivative $\frac{dy}{dx}$ at $x = -1$.



[2]

3 In the list of ten matrices, the inverse of each matrix is one of the other nine matrices. [5]

Match each matrix with its inverse. (No explanation is needed.)

$$M_1 = \begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & -0.5 & 0 & 0 \\ 0 & 0 & 0.25 & 0 \\ 0 & 0 & 0 & -0.2 \end{bmatrix} \qquad M_2 = \begin{bmatrix} 7 & 9 \\ 6 & 8 \end{bmatrix}$$

$$M_3 = \begin{bmatrix} -0.4 & 2.7 & -3.7 \\ 0.4 & 2.3 & -3.3 \\ 0 & 3 & -5 \end{bmatrix} \qquad M_4 = \begin{bmatrix} 10 & 0 \\ 0 & -1 \end{bmatrix}$$

$$M_5 = \begin{bmatrix} 27 & -10 & -13 \\ -8 & 3 & 4 \\ -72 & 27 & 35 \end{bmatrix} \qquad M_6 = \begin{bmatrix} 4.0 & -4.5 \\ -3.0 & 3.5 \end{bmatrix}$$

$$M_7 = \begin{bmatrix} 0.2 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & -5 \end{bmatrix} \qquad M_8 = \begin{bmatrix} 0.1 & 0 \\ 0 & -10 \end{bmatrix}$$

$$M_9 = \begin{bmatrix} -1 & 1.5 & -0.25 \\ 1.25 & 1.25 & -1.75 \\ 0.75 & 0.75 & -1.25 \end{bmatrix} \qquad M_{10} = \begin{bmatrix} 3 & 1 & 1 \\ 8 & -9 & 4 \\ 0 & 9 & -1 \end{bmatrix}$$

4 a) Evaluate the integral:

$$I = \int \left(x - \frac{1}{x^2} \right) dx$$

[3]

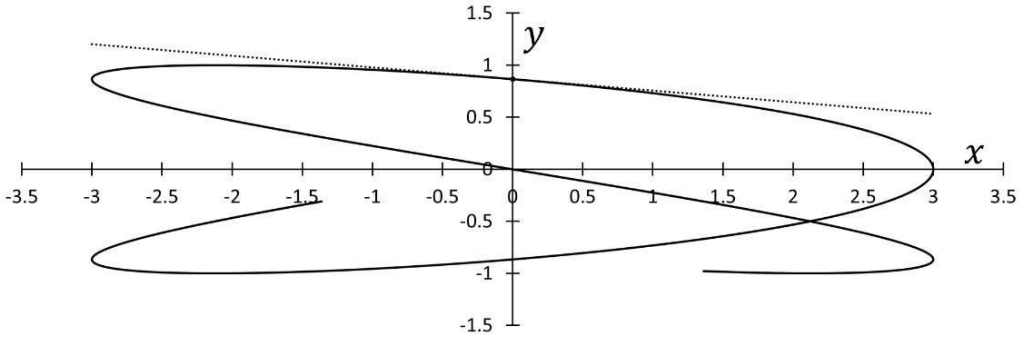
b) Check the answer by differentiation.

[2]

5 The (x, y) coordinates of the graph below are defined by parameter t:

$$x = 3\cos\left(\frac{\pi}{2}t\right)$$

$$y = \sin\left(\frac{\pi}{3}t\right)$$



a) Determine the (x, y) coordinates of the graph for parameter $t = 1$;

[2]

b) Work out the derivative of y with respect to x at $t = 1$ to find the gradient of the (x, y) graph at point with parameter value $t = 1$.

[3]

6 By evaluating the determinants of matrices, A, B, and C, where: [5]

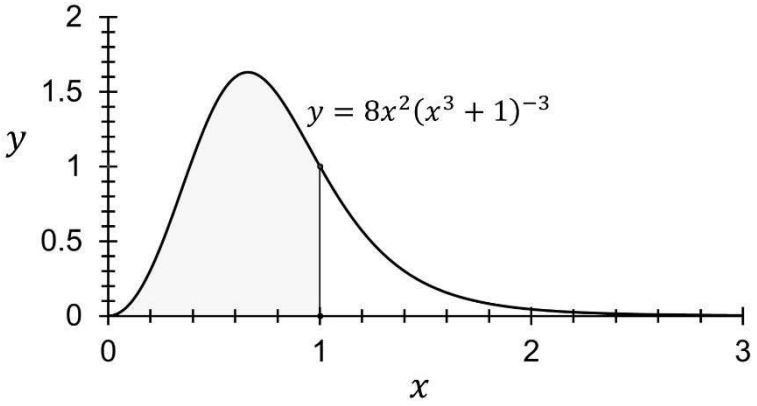
$$A = \begin{bmatrix} 0 & 4 \\ 2 & 3 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & -6 \\ 2 & -1 \end{bmatrix} \qquad C = AB$$

Verify the identity: $\det(A)\det(B) \equiv \det(AB)$

7 Using the substitution technique of integration, evaluate [5]

:

$$T = \int_0^1 8x^2(x^3+1)^{-3}dx$$



8 a) Find y that satisfies the equation for $x = -2$:

$$x^3y + xy^3 = 54 - 8y$$

[1]

b) Work out the derivative: $\frac{dy}{dx}$

[3]

c) Evaluate the derivative $\frac{dy}{dx}$ at $x = -2$. Simplify the answer if possible.

[1]

Section B

Answer TWO questions.

9 a) Two elements of a geometric sequence are:

$$a_2 = 54 \quad a_5 = -16$$

i) Determine the common ratio of the sequence. (Simplify.)

[2]

ii) Derive the first element, a_1 of the sequence.

[1]

iii) Find the sum of the first five elements:

$$S_5 \equiv \sum_{i=1}^{5} a_i \equiv a_1 + a_2 + a_3 + a_4$$

[4]

iv) Show that the geometric series:

$$S \equiv \sum_{i=1}^{\infty} a_i \equiv a_1 + a_2 + \cdots$$

… exists.

[1]

v) Evaluate the series, S.

[2]

b) Using partial fractions, evaluate the integral:

$$P = \int_0^2 \frac{3x + 1}{x^2 + 4x + 3}\, dx$$

[10]

10 a) A, B, C, D, E, and F are six matrices:

$$A = \begin{bmatrix} -6 & 9 \\ 2 & -3 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 8 & 3 & 0 & -9 \\ -3 & -2 & 1 & 6 & -7 \end{bmatrix} \quad C = \begin{bmatrix} 1 & -4 & 5 & -5 \\ 0 & -4 & 1 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 0 \\ 3 & 1 \\ -2 & 4 \end{bmatrix} \quad E = \begin{bmatrix} 5 & 5 \\ 1 & 2 \\ 4 & -2 \\ 4 & 3 \end{bmatrix} \quad F = \begin{bmatrix} -7 \\ -1 \end{bmatrix}$$

i) Select two of them so that their product equals a 2×2 matrix, M.

[3]

ii) Work out the determinant, $|M|$, and inverse, M^{-1}, of the product matrix, M.

[3]

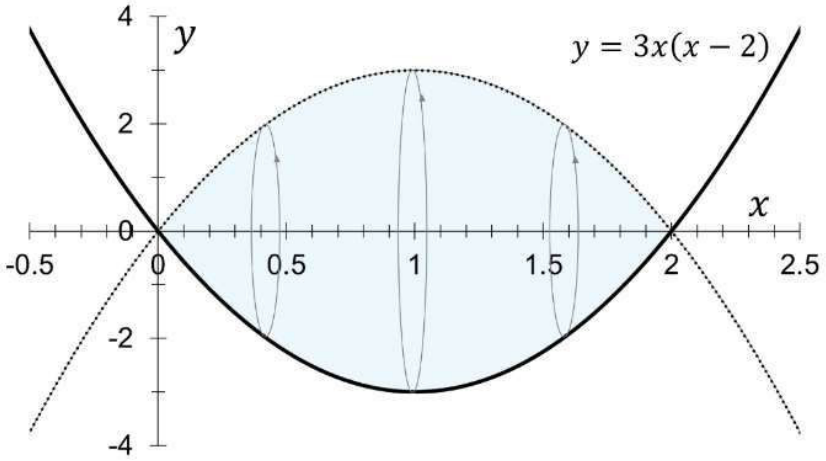
iii) By multiplication, check that the two matrices obtained in (i) and (ii) are indeed inverse of each other.

[1]

iv) Evaluate $N = 5(M^{-1} - M)$

[3]

b) Determine the volume, V, of the shape obtained by rotating the graph $y = 3x(x - 2)$ about the x axis between $x_1 = 0$ and $x_2 = 2$.



[10]

11 a) Using trigonometric identities

i) Knowing that $\sin(37°) \approx 0.6$, find an approximate value for $\cos(37°)$

[2]

ii) Using trigonometric identities, derive an approximate value for $\cot(53°)$

[3]

iii) Find a solutions to the quadratic trigonometric equation for $-5 < x < 0$:

$$2\cos^2 x + 5\sin x = 5$$

[5]

b) In the binomial expansion of:

$$B = \left(2x^2 + \frac{\sqrt{x}}{2}\right)^7$$

Find the coefficient of the x^8 term.

[10]

END OF PAPER