PRIFYSGOL ABERYSTWYTH UNIVERSITY

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DEPARTMENT OF PHYSICS

SEMESTER 2 EXAMINATIONS, MAY 2026

PH22510 – Electricity and Magnetism

Time allowed – 2 hours

- **Full marks can be attained for complete answers to ALL six questions in Section A and TWO questions from Section B**
- **The numbers in brackets after the questions indicate the allocation of marks for each question.**
- **Only calculators from the Casio FX83, FX85 or FX991 range may be used.**
- **A booklet of mathematical formulae and standard constants will be provided.**
- **Only a 1 side of A4 summary sheet is allowed.**
- **MAE MODD I FYFYRWYR GYFLWYNO ATEBION I'R PAPUR HWN NAILL AI YN Y GYMRAEG NEU'R SAESNEG**
- **STUDENTS MAY SUBMIT ANSWERS TO THIS PAPER IN EITHER WELSH OR ENGLISH**

Once seated, students may complete the front cover of the answer book(s) and the attendance slip ONLY.

DO NOT TURN THE EXAMINATION QUESTION PAPER OVER UNTIL INSTRUCTED TO DO SO.

Section A

Attempt ALL questions in the booklet provided

1 In a region, the electric field is parallel to the x-axis and its magnitude can be written as $E = 2x$ V/m. Find the electrostatic potential difference between point A with coordinates (2,5,0) and point B with coordinates (4,0,0). Which point has a higher potential? [5]

2 In electrostatics, a piece of metal is surrounded by a dielectric material. Explain why the electric field in the dielectric material at the boundary to the metal is perpendicular to the boundary. You may want to use the boundary condition of the electric field and the properties of a metal. [5]

3 The electric displacement field in a dielectric material with a relative permittivity ε in Cartesian coordinate system can be written as [5]

$$\mathbf{D} = D_0 \left(\frac{x}{a}\right)^2 \mathbf{e}_x$$

where D_0 and a are constants and $\mathbf{e}_x$ is a unit vector along the x-axis. Find the expressions of the free volume charge density and the polarization field.

4 A proton moves in a uniform magnetic field [5]

$$\mathbf{B} = (0.0, 0.0, 4.0) \times 10^{-3}\ \mathrm{T}$$

At time $t = 0$ the proton has an instantaneous velocity

$$\mathbf{v} = (0.0, 1.5, 0.7) \times 10^5\ \mathrm{m\ s^{-1}}$$

i) Determine the magnetic force on the proton at time $t = 0$, showing your work clearly.

ii) Describe the motion of the proton after $t = 0$.

iii) In a different experiment an alpha particle is used, without further calculation describe the motion in the same field as used previously.

5 A rectangular wire loop ABCD is positioned at a distance of 1.2 cm from a long straight wire that carries a current $I = 3.0\ \mathrm{A}$, as shown in the diagram. Side AB is 1.2 cm and side BC is 5.0 cm. The loop is moved vertically upward and after 0.8 s the effect of the current I on the loop is negligible. By considering the initial and final magnetic flux through the loop, determine the average electromotive force (e.m.f.) induced in the loop. [5]

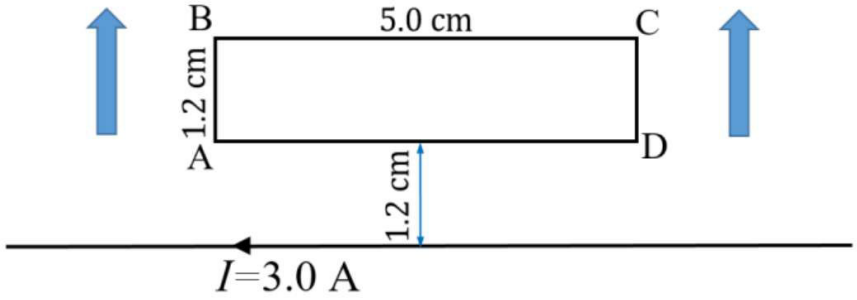


Figure 1: Diagram of the setup a rectangular coil ABCD and a current carrying long straight wire.

6 State Gauss' law for magnetism in differential and integral forms. Explain how Gauss' law for magnetism is equivalent to the statement 'there are no magnetic monopoles'. [5]

Section B

Answer TWO questions

7 A spherical capacitor consists of two concentric, conducting spherical shells of radius R_1 (inner) and R_2 (outer), storing equal and opposite charges Q (on the inner shell) and $-Q$ (on the outer shell). The outer shell is earthed so its potential is at 0 V.

a) Using physics principles (for example, Coulomb's law), argue that electric field between the shells has only a radial component. Also argue that the electric field only depends on the distance to the centre of the capacitor r.

[5]

b) Use Gauss' law to find an expression for the electric field between the shells.

[5]

c) Find the expression for the electric potential at the inner shell.

[5]

8 a) A toroidal (doughnut shaped) sample of ferromagnetic material with relative permeability μ has a mean radius R. The toroid is cut into two halves and the parts separated by a small distance ℓ. The distance ℓ is small compared to the circumference of the torus. A single wire is overwound on the two halves and around the air gaps separating them, with a total of N_t turns. The setup is shown in Figure 2 (on the next page).

If the wire carries current I, show that the magnetic field in the material of this electromagnet is:

$$B = \frac{\mu_0 \mu N_t I}{2(\pi R + \mu \ell)}$$

In your answer indicate clearly the physics and any assumptions considered in the derivation. Consider all relevant Maxwell equations and the field conditions at boundaries between media.

[5]

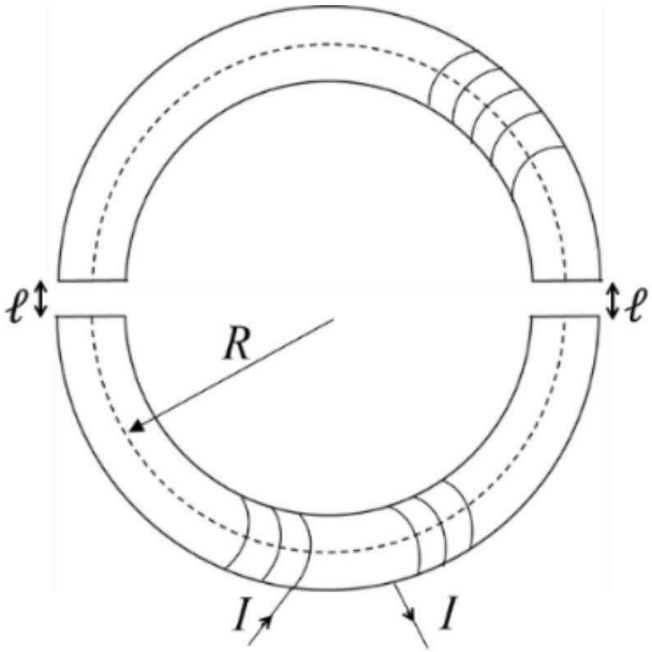


Figure 2: Diagram of a toroidal electromagnet with two air gaps.

b) Knowing that the relative permeability μ of the material of the toroid is 2000, the mean radius R of the original un-cut toroid is 10 cm, the cross-sectional radius is 6mm, the total number of turns N_t is 1600, the current I is 2 A and the length ℓ of each gap is 3 mm, determine:

i) the magnitude of the magnetic field,

[2]

ii) the magnitude of the magnetic intensity inside the material,

[2]

iii) the magnetic flux through all turns of the wire.

[2]

c) A student says that the magnetic field at a radial distance larger than that of

the outer radius of the toroid is zero. State whether this statement is correct, justifying your answer.

[4]

9 An infinitely large horizontal metal plane (which is located in the $z = 0$ plane) is earthed. A point charge Q is placed above the plane and its coordinates are (0, 0, a) .

a) The image charge method in electrostatics is based on the *uniqueness theorem*. Describe the meaning of the uniqueness theorem and how this leads to the use of image charge method in some applications.

[5]

b) Suppose the distance between an arbitrary point P on the metal plane and the origin is r. Show that the magnitude of the electric field at point P can be written as

$$E = \frac{Q}{2\pi\varepsilon_0}\frac{a}{(a^2 + r^2)^{3/2}}$$

[5]

c) What is the induced surface charge density (as an expression) at point P?

[1]

d) By using the surface charge density from c) and integrating over all the metal plane, show that the total induced surface charges on the plane is equal to $-Q$. It is also fine if you use an alternative method.

[4]

END OF PAPER