PRIFYSGOL ABERYSTWYTH UNIVERSITY

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DEPARTMENT OF PHYSICS

SEMESTER 2 EXAMINATIONS, MAY 2026

PH33020 – Particles, Quanta and Fields

Time allowed – 3 hours

- **Full marks can be attained for complete answers to ALL eight questions in Section A and TWO questions from Section B**
- **The numbers in brackets after the questions indicate the allocation of marks for each question.**
- **Only calculators from the Casio FX83, FX85 or FX991 range may be used.**
- **A booklet of mathematical formulae and standard constants will be provided.**
- **Only a 1 side of A4 summary sheet is allowed.**
- **MAE MODD I FYFYRWYR GYFLWYNO ATEBION I'R PAPUR HWN NAILL AI YN Y GYMRAEG NEU'R SAESNEG**
- **STUDENTS MAY SUBMIT ANSWERS TO THIS PAPER IN EITHER WELSH OR ENGLISH**

Once seated, students may complete the front cover of the answer book(s) and the attendance slip ONLY.

DO NOT TURN THE EXAMINATION QUESTION PAPER OVER UNTIL INSTRUCTED TO DO SO.

Section A

Attempt ALL questions in the booklet provided

1 Write down the formula for the expectation value $\langle \hat{A} \rangle$ of an operator $\hat{A}$, given a one-dimensional wavefunction $\psi(x)$. Express this both in Dirac notation and as an integral. [5]

2 Explain the difference between Fermions and Bosons in terms of the spin quantum numbers s and m_s. What happens to the wavefunction when two electrons are interchanged? [5]

3 Consider two electrons in an atom, where l_1 represents the orbital angular momentum quantum number for electron 1, while l_2 represents the orbital angular momentum quantum number for electron 2. State the general rule for the allowed values of the combined quantum number j, representing the sum of the angular momentum of the two electrons. You can ignore spin. [5]

What are the allowed values for j when $l_1 = 2$ and $l_2 = 1$?

4 The parity transformation is based around the following change of the position vector: [5]

$$\boldsymbol{r} \rightarrow -\boldsymbol{r}\,.$$

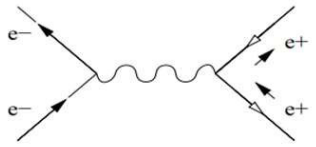
How should true vectors transform under parity transformation?

The angular momentum $\boldsymbol{J} = \boldsymbol{r} \times \boldsymbol{p}$ is instead an axial vector: show that it does not transform as a true vector. Give another example of an axial vector occurring in Physics.

[5]

5 Define the *helicity* of a particle. Explain why this property can be Lorentz invariant only for particles that travel at the speed of light. [5]

6 The picture below shows a two-vertex QED Feynman diagram for Bhabha scattering. [5]



Draw the second two-vertex diagram for this process. Sketch a higher order contribution.

7 The decay of the muon was observed by E.J. Williams in 1940. The process was described as [5]

$$\mu^- \rightarrow e^- + (\text{unknown}).$$

At the time, the other particles created in the decay were unknown, however, they are now known to be neutrinos. State what specific type of neutrinos they must be, justifying your answer with the appropriate conservation law.

8 [5]

In the particle decay process $\Sigma^+ \longrightarrow p^+ + \pi^0$, we find that the total strangeness is not conserved. Which interaction should be behind this decay? Explain your reasoning.

Section B

Answer TWO questions

9 Consider the particle in an infinite well of size L:

$$V(x) = \infty \quad if\ x < 0$$

$$V(x) = 0 \quad if\ 0 \leq x \leq L$$

$$V(x) = \infty \quad if\ x > L$$

The purpose of this question is to estimate the ground state energy of the particle in an infinite well using the *variational principle*. The trial wavefunction $v(x)$ is given by:

$$v(x) = Nx(L - x) \quad if\ 0 \leq x \leq L$$

$$v(x) = 0 \qquad\qquad elsewhere$$

where L is the size of the infinite well and N is a normalisation constant.

a) Show that:

$$\langle v|v\rangle = \int_0^L |v(x)|^2 dx = \frac{N^2 L^5}{30}$$

and hence show that the normalisation constant N is given by:

$$N = \sqrt{\frac{30}{L^5}}$$

[7]

b) Show that

$$\left\langle v\left|\frac{d^2 v}{dx^2}\right.\right\rangle = \int_0^L v(x)^* \frac{d^2 v(x)}{dx^2} dx = -\frac{10}{L^2}$$

and hence

$$\langle v|\hat{H}|v\rangle = -\frac{\hbar^2}{2m}\left\langle v\left|\frac{d^2 v}{dx^2}\right.\right\rangle = \frac{5\hbar^2}{mL^2}$$

[8]

c) State the exact energy levels for the particle in an infinite well of size L. What is the ground state energy for this system? How does this compare to the result from b)?

[5]

10 A system consists of N identical quantum particles and has the wavefunction

$$\Psi = \Psi(x_1, x_2, \cdots, x_N).$$

Let σ be a permutation of the indices: that is, $\sigma(1), \cdots, \sigma(N)$ lists all the indices $1, \cdots, N$ though not necessarily in the original order.

a) The wavefunction has the symmetry

$$\Psi\left(x_{\sigma(1)}, \cdots, x_{\sigma(N)}\right) = \xi\, \Psi(x_1, \cdots, x_N),$$

where ξ is a scalar depending on the permutation σ.

State the appropriate values for ξ for bosons and for fermions.

[4]

b) What are the implications of the above symmetries for the probability density $|\Psi|^2$.

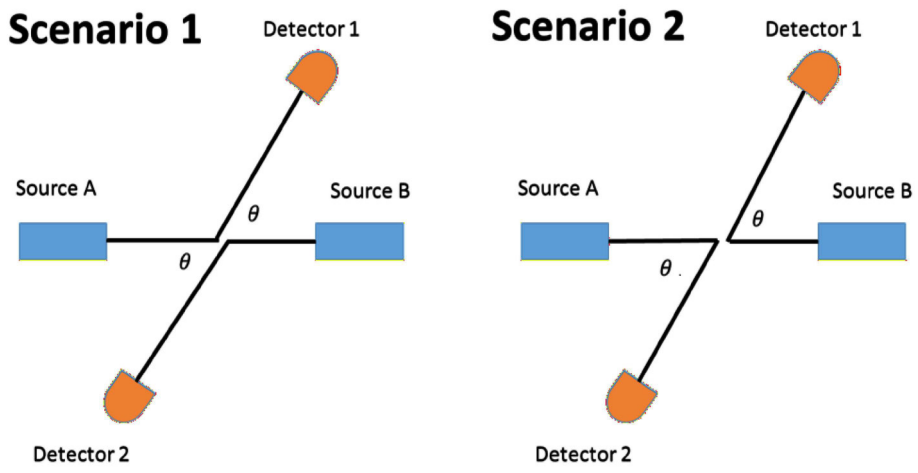
[2]

c) A gas of non-interacting identical bosons has one-particle states $\phi_0, \phi_1, \phi_2, \cdots$. Write down the wavefunction for $N = 3$ bosons when two of them are in the ground state ϕ_0 and one is in the first excited state ϕ_1.

[5]

d) How does this change if the particles were non-interacting fermions?

e) Two quantum particles scatter through an angle θ: classically we identify two scenarios as shown below depending on whether the particle from source A goes to detector 1 and the particle from source B goes to detector 2, or *vice versa*.



Let $a(\theta)$ be the probability amplitude associated with the scenario on the left, so that $a(\pi - \theta)$ is the probability amplitude for the scenario on the right.

Calculate the overall probability for the two particles to scatter through an angle θ (by either scenario) when the particles are

i. Distinguishable,
ii. Identical bosons,
iii. Identical fermions.

[6]

f) How are these probabilities related when $\theta = \frac{\pi}{2}$?

[3]

11 a) The thermodynamic potential $\Psi = \Psi(\mu, T, V)$ is defined as $\Psi = U - TS - \mu N$. Assuming the identity $TdS = dU + pdV - \mu dN$, show that

$$S = -\frac{\partial \Psi}{\partial T}, \quad N = -\frac{\partial \Psi}{\partial \mu}, \quad p = -\frac{\partial \Psi}{\partial V}.$$

[5]

b) For a dilute gas, one may assume that Ψ is proportional to the volume V, that is, $\Psi(\mu, T, V) = c(\mu, T)V$. Show that the pressure is then given by

$$p = -\Psi / V.$$

[5]

c) In the grand canonical ensemble, the probability associated with a microstate x takes the form

$$p(x) = \frac{1}{\Xi} e^{-\beta[E(x) - \mu N(x)]}$$

where $N(x)$ and $E(x)$ are the number of particles and the energy associated with the microstate and Ξ is the grand canonical partition function. The Shannon entropy is

$$H = -\sum_x p(x) \log p(x).$$

Compute H for the grand canonical ensemble and taking the entropy to be $S = k\,H$ show that we should identify β as $1/kT$, μ as the chemical potential, and $\Psi = -kT \log \Xi$.

[5]

d) For an ideal gas of fermions, take the energy eigenvalue ϵ_k for the kth one-particle energy eigenstate. The microstates x are the occupation numbers $(n_0, n_1, n_2, \cdots)$ where n_k counts the number of fermion particles occupying the kth energy eigenstate. From this, show that the pressure is given by

$$p = \frac{1}{\beta V} \sum_k \log\left(1 + e^{-\beta(\epsilon_k - \mu)}\right).$$

[5]

END OF PAPER