PRIFYSGOL ABERYSTWYTH UNIVERSITY

ADRAN FFISEG

DEPARTMENT OF PHYSICS

SEMESTER 2 EXAMINATIONS, MAY 2026

PH39820 – ASTROPHYSICS II

Time allowed – 3 hours

- **Full marks can be attained for complete answers to ALL eight questions in Section A and TWO questions from Section B**
- **The numbers in brackets after the questions indicate the allocation of marks for each question.**
- **Only calculators from the Casio FX83, FX85 or FX991 range may be used.**
- **A booklet of mathematical formulae and standard constants will be provided.**
- **Only a 1 side of A4 summary sheet is allowed.**
- **MAE MODD I FYFYRWYR GYFLWYNO ATEBION I'R PAPUR HWN NAILL AI YN Y GYMRAEG NEU'R SAESNEG**
- **STUDENTS MAY SUBMIT ANSWERS TO THIS PAPER IN EITHER WELSH OR ENGLISH**

Once seated, students may complete the front cover of the answer book(s) and the attendance slip ONLY.

DO NOT TURN THE EXAMINATION QUESTION PAPER OVER UNTIL INSTRUCTED TO DO SO.

Section A

Attempt ALL questions in the booklet provided

1 The radial velocity V_r of a star in the Milky Way relative to the Sun is given by: [5]

$$V_r = R_0 \sin l \left(\frac{V}{R} - \frac{V_0}{R_0}\right),$$

where l is the Galactic longitude, V and R are the rotational velocity and the Galactocentric distance of the star, and V_0 and R_0 are the corresponding values for the Sun.

a) If the Milky Way rotated as a rigid body, determine and describe the relative motion of stars relative to the Sun using the given equation.

b) Assuming $0° < l < 90°$ and a flat rotation curve, use the given equation to find the sign of V_r for nearby stars with $R < R_0$ and for distant stars with $R > R_0$.

2 Answer true or false to the following statements: [5]

a) Stellar interactions generally play a more significant role in globular clusters, where star densities and velocities are higher compared to those in the galactic disk.

b) Weak encounters between stars typically produce small cumulative changes in stellar velocities rather than dramatic orbital alterations in a single event.

c) Strong encounters between stars are far more common than weak encounters in most stellar systems.

d) Spiral arms in galaxies are thought to be long-lived density waves that move through the disk rather than groups of stars that remain permanently together.

e) Stars and gas in a spiral galaxy rotate at the same angular speed as the spiral arms at all radii.

3 The distribution of surface brightness in elliptical galaxies is given by the following formula: [5]

$$I(R) = I(0)\exp\left[-2n\left(\frac{R}{R_e}\right)^{\frac{1}{n}}\right],$$

where $I(0)$ is the central brightness, R_e is the effective radius, and n is a natural number. Explore the behaviour of the surface brightness at large R to explain why lower values of n characterise small-sized ellipticals, whereas larger values of n are indicative of giant ellipticals.

4 Outline the arguments that provide a physically plausible basis for the Tully-Fisher relationship. [5]

5 The components of covariant vectors A^α and B_β in reference frame S at point P in a two-dimensional space are: [5]

$$A^\alpha = [4, -7] \text{ and } B_\beta = [5, 1].$$

The coordinates of reference frame $\tilde{\mathrm{S}}$ are: $\tilde{x}^1 = x^2 - 2x^1$ and $\tilde{x}^2 = 0.5x^2$.

(Therefore, the coordinates of S are: $x^1 = \tilde{x}^2 - 0.5\tilde{x}^1$ and $x^2 = 2\tilde{x}^2$)

By working out the scalar product of vectors A^α and B_β, in S, and the scalar product of vectors $\tilde{A}^\alpha$ and $\tilde{B}_\beta$, in $\tilde{\mathrm{S}}$, verify that the scalar product of covariant and contravariant vectors is a scalar.

The transformation formulas for vectors are:

$$\tilde{A}^\alpha = \frac{\partial \tilde{x}^\alpha}{\partial x^\lambda} A^\lambda \quad \text{and} \quad \tilde{B}_\beta = \frac{\partial x^\beta}{\partial \tilde{x}^\lambda} B_\lambda$$

6 In a three-dimensional space, with coordinates in reference frame S, at point P $(x_\mathrm{P}, y_\mathrm{P}, z_\mathrm{P}) = [3.1, 4.1, 5.9]$, the affine connection is: [5]

$$\Gamma^\lambda_{\alpha\beta} = \left[\begin{array}{ccc|ccc|ccc} \Gamma^1_{11} & \Gamma^1_{12} & \Gamma^1_{13} & \Gamma^2_{11} & \Gamma^2_{12} & \Gamma^2_{13} & \Gamma^3_{11} & \Gamma^3_{12} & \Gamma^3_{13} \\ \Gamma^1_{21} & \Gamma^1_{22} & \Gamma^1_{23} & \Gamma^2_{21} & \Gamma^2_{22} & \Gamma^2_{23} & \Gamma^{31}_{21} & \Gamma^3_{22} & \Gamma^3_{23} \\ \Gamma^1_{31} & \Gamma^1_{32} & \Gamma^1_{33} & \Gamma^2_{31} & \Gamma^2_{32} & \Gamma^2_{33} & \Gamma^3_{31} & \Gamma^3_{32} & \Gamma^3_{33} \end{array}\right]$$

$$= \left[\begin{array}{ccc|ccc|ccc} 1 & 2 & 3 & 4 & 0 & 6 & 7 & 2 & 5 \\ 2 & 3 & 4 & 6 & 6 & 7 & 8 & 9 & 8 \\ 1 & 0 & 1 & 2 & 3 & 4 & 9 & 6 & 7 \end{array}\right]$$

Give the elements $\Gamma^{\lambda}_{[13]}$ of the torsion tensor, $\Gamma^{\lambda}_{[\alpha\beta]}$, at point P.

7 Answer True or False to the following statements regarding modern cosmology. [5]

a) A fundamental observer is someone who does observations fundamentally correct all the time.

b) Since the Apollo mission, none of the numerous space missions have found evidence of dark matter in the solar system. Hence we can conclude that dark matter does not exist in the solar system.

c) The expansion of the universe is so powerful that eventually our Milky Way galaxy will be torn apart.

d) Nothing can move faster than photons according to Einstein's special relativity. Hence the speed of distant galaxies observed by us can not be larger than the speed of light in free space.

e) Heat flows from high temperature regions to low temperature regions. Hence the universe eventually will reach a thermal equilibrium, or 'thermal death'.

8 Explain the fine-tuning problem and horizon problem of the big bang theory. [5]

Section B

Answer TWO questions

9 This question is concerned with active galactic nuclei.

a) List three important observational pieces of evidence which lend support to the existence of black holes at the centre of active galactic nuclei.

[5]

b) The spectrum from a quasar shows a prominent peak at 121.6 nm corresponding to Ly-α. The width of this line is around 4 nm. Estimate the average speed of the emitting gas along the line of sight.

[4]

c) Assume that the speed obtained in b) is due to the rotation of gas around the central nucleus. Estimate the mass contained within the orbit if it has a radius of 0.01 pc.

[6]

d) Find the Schwarzchild radius corresponding to the mass obtained in c). [5]

10 a) The metric in a region in a two-dimensional space is:

$$ds^2 = x^2(dx^1)^2 + 2(dx^1)(dx^2)$$

i) Give the metric tensor, $g_{\alpha\beta}$, in the region.

[2]

ii) Find the contravariant metric tensor, $g^{\alpha\beta}$.

[3]

iii) Work out element Γ^2_{12} of the Christoffel symbol.

$$\Gamma^{\rho}_{\lambda\mu} = \frac{1}{2} g^{\rho\kappa}\left(g_{\kappa\mu,\lambda} + g_{\lambda\kappa,\mu} - g_{\lambda\mu,\kappa}\right)$$

[5]

b) The Schwarzschild solution to the field equation is:

$$ds^2 = -\left(1 - \frac{R_g}{r}\right)(dt)^2 + \left(1 - \frac{R_g}{r}\right)^{-1}(dr)^2 + r^2(d\theta)^2 + r^2 \sin^2\theta\,(d\phi)^2$$

i) What assumption is made about the star, when the above metric, Schwarzschild metric, is used to describe the geometry of space-time around the star?

[2]

ii) Explain the physical meaning of the Schwarzschild radius (or gravitational radius), R_{g}.

[2]

iii) Which physical property of the star can be derived from knowing the above metric?

[2]

iv) State Birkhoff's theorem and explain its consequence concerning astronomical observations.

[4]

11 a) Explain the origin of the cosmic microwave background (CMB) radiation.

[5]

b) Explain why the extremely small temperature fluctuations of CMB suggest that dark matter must be present.

[6]

c) Hubble's law is given as

$$v = H_0 d$$

i) Explain the meaning of the symbols in the equation. Using simple physical arguments explain why H_0 will decrease in future.

[5]

ii) The best current observations give the value of H_0 as 68 km s^{-1} Mpc^{-1}. Find the value of *Hubble time* (E.g. in Gyr).

[4]

END OF PAPER