

MATHEMATICS

Time allowed: 1 hour 30 minutes

- All answers (including any diagrams, graphs or sketches) should be written on paper, and scanned into a **single** PDF file. Graph paper is not required.
 - Answer **all** questions in Section A and **two** questions from Section B.
 - Candidates are permitted to use calculators, provided they comply with A level examining board regulations. They must be made available on request for inspection by invigilators, who are authorised to remove any suspect calculators.
 - Statistical tables are not required.
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Section A

1. Simplify the following expressions as far as possible, showing your working clearly.

(a) $1 + \sqrt{(1 - a)^2}$ assuming $a > 1$; [2 marks]

(b) $\frac{\sqrt{40}}{5}$; [4 marks]
 $\frac{3 - \sqrt{8}}{-15}$

(c) $4 \log_3 15 + 2 \log_3 4 - \log_3 10000$. [5 marks]

2. Solve the following equations for x :

(a) $243^x = 9$; [2 marks]

(b) $9^x - 10 \times 3^x + 9 = 0$. [5 marks]

3. Solve the equation

$$(15 \cos \theta - 7 \tan \theta) \cos \theta = 13$$

for values of θ between 0° and 180° . [7 marks]

4. In the binomial expansion of $(1 + ax)^8$ in powers of x , the coefficient of x^5 is equal to $-\frac{56}{243}$. Find the value of a . [4 marks]

5. Find the range of values of x for which the function $f(x) = x^3 - 3x^2 + 2x - 1$ is increasing. [7 marks]

6. Consider the points $A(1, 2)$ and $B(-2, 3)$ in the xy -plane. Find the equation of the line perpendicular to AB and passing through the midpoint of AB . [5 marks]

7. In the following statements A and B, x and y are real numbers.

A If $(x - y)^2 = (x + y)^2$, then $x = y = 0$.

B If $(x - y)^2 = -(x + y)^2$, then $x = y = 0$.

In both cases, identify whether the statement is true or false. Justify your answer by giving a proof (if true), or a counterexample (if false). [5 marks]

8. Consider the function $f(x) = 6x^2 - 7x + 2$ (defined for all real x).

(a) Determine the function $F(x)$ for which $F'(x) = f(x)$ and $F(0) = 1$. [3 marks]

(b) Find the equation of the line in the xy -plane tangent to the curve $y = F(x)$ at $x = 0$. [4 marks]

9. Points A and B have position vectors $\mathbf{a} = \mathbf{i} - 5\mathbf{j}$ and $\mathbf{b} = -3\mathbf{i} + \mathbf{j}$.

(a) Find the vector \mathbf{AB} (in terms of \mathbf{i} and \mathbf{j}), and determine whether it is parallel to the vector $2\mathbf{i} - 3\mathbf{j}$, explaining your reason. [4 marks]

(b) Point C divides the line AB such that $AC : CB = 2 : 1$. Find, in terms of \mathbf{i} and \mathbf{j} , the position vector of C . [3 marks]

Section B

10. Circle C_1 has equation

$$x^2 + y^2 + 4(y - x) - 17 = 0.$$

Circle C_2 is centred at point $(-4, 6)$ and has radius 5.

- (a) Find the radius and the coordinates of the centre of C_1 . [6 marks]
- (b) Find the points at which C_2 intersects the line $y = x + 9$. [6 marks]
- (c) Show that C_1 and C_2 touch at exactly one point P , and find
 - (i) the coordinates of P ; [4 marks]
 - (ii) the equation of the line tangent to the circles at P . [4 marks]

11. Two weights A and B , of masses $m_A = 3\text{ kg}$ and $m_B = 6\text{ kg}$, are connected to each other by a vertical uniform rod R of mass m_R , as shown in the figure. An upward force $F = 100\text{ N}$ is exerted on A , and the whole system falls downwards with acceleration $a = 2\text{ ms}^{-2}$.

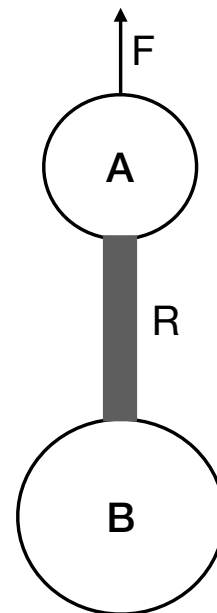
- (a) Sketch free-body diagrams for the following objects:
 - (i) The system consisting of A , B , and R ; [1 mark]
 - (ii) The weight A ; [1 mark]
 - (iii) The weight B . [1 mark]

In each case, indicate all the forces acting on the object. (Numerical magnitudes are not required at this point.)

- (b) Using the diagrams in part (a), find (to 2dp)
 - (i) the mass m_R ; [4 marks]
 - (ii) the tension force at the top of R ; [4 marks]
 - (iii) the tension force at the bottom of R . [3 marks]

Make sure to show sufficient working, referring to the relevant law of motion. Acceleration due to gravity should be taken as $g = 9.81\text{ ms}^{-2}$.

- (c) Suppose that the length of R is 30 cm. Point P lies on R at distance 10 cm from the bottom.
 - (i) Sketch a free-body diagram for the part of the rod below P . [1 mark]
 - (ii) Find (to 2dp) the tension force at P . [5 marks]

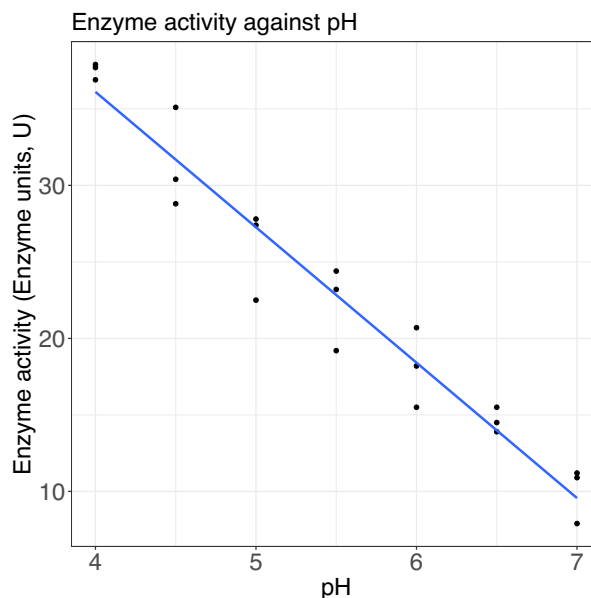


12. (a) In a single round of a certain game it is possible to score 1, 2, 3, 4, or 5 points. The probabilities for these scores are given in the following table, where a and b are constants:

score	1	2	3	4	5
probability	0.2	a	0.1	b	0.15

Furthermore, the probability of scoring less than 4 points is equal to the probability of scoring at least 4 points.

- (i) Determine the values of a and b . [4 marks]
- (ii) Each round of the game is independent of any previous rounds. Jemima plays two rounds and adds the two scores together to get a total. Find the probability that her total is greater than 8. [5 marks]
- (b) In a biological experiment, the activity of a particular enzyme (in enzyme units, U), was measured at a range of pH values. The graph of enzyme activity against pH is shown below.



- (i) Describe the relationship between pH and enzyme activity. [3 marks]
- (ii) The equation of the regression line is

$$\text{Enzyme activity} = 71.5 - 8.85 \text{ pH.}$$

What do each of the numbers in this equation correspond to? [2 marks]

- (iii) What is the predicted enzyme activity at a pH of 6.2? [2 marks]
- (iv) What is the problem with using this equation to predict the enzyme activity at a pH of 9? What does this tell you about the relationship between enzyme activity and pH over the range 4 to 9? [4 marks]