

MATHEMATICS

Time allowed: 1 hour 30 minutes

- All answers (including any diagrams, graphs or sketches) should be written on paper, and scanned into a **single** PDF file. Graph paper is not required.
 - Answer **all** questions in Section A and **two** questions from Section B.
 - Candidates are permitted to use calculators, provided they comply with A level examining board regulations. They must be made available on request for inspection by invigilators, who are authorised to remove any suspect calculators.
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Information

- Acceleration due to gravity should be taken to be 9.81 ms^{-2} .
- Statistical tables will be provided. Note that the tables refer to the **right-hand** tails of the distributions, that is, probabilities of the form $p = \mathbb{P}(X \geq x)$ where X is a random variable and x an **upper** percentage point of its distribution.
- Formulas related to standard distributions (e.g. for probability, mean, and variance) can be found on the back page of the statistical tables.

Section A

1. Simplify the following expressions as far as possible, showing your working clearly.

(a) $\frac{1}{1 - \frac{1}{1+x}} - \frac{1-x}{x}$; [3 marks]

(b) $\frac{2\sqrt{5}}{3 + \sqrt{5}} - \frac{2\sqrt{5}}{3 - \sqrt{5}}$; [3 marks]

(c) $\log_2(2x^2 - 4x - 30) - \log_2(x + 3) - \log_2(x - 5)$, where $x > 5$. [5 marks]

2. Solve (to 3 dp) the following equations for x :

(a) $e^{-2x} = 7$; [2 marks]

(b) $4^x - 7 \times 2^x - 3 = 0$. [6 marks]

3. Show that $\sqrt{103 + 20\sqrt{3}} = 10 + \sqrt{3}$. [2 marks]

4. Without using any decimal approximations, solve the equation

$$2 \cos^2 \theta - 9 \sin \theta + 3 = 0$$

for values of θ between 0° and 180° . [7 marks]

5. In the binomial expansion of $(x^2 - a)^9$ in powers of x , the coefficient of x^8 is equal to $-\frac{14}{27}$. Find the value of a . [4 marks]

6. Find the range of values of x for which $x^2 + 5 \geq 2x(x - 2)$. [5 marks]

7. A curve C has equation $x^2 + y^2 - x - \frac{2}{3}y + \frac{1}{3} = 0$. Show that C is a circle and find its centre and radius. [6 marks]

8. In the following statements A and B, x and y are real numbers.

A If $x^2 + x = 2$, then $x = 1$.

B If $x^2 + x = 6$ and $x > 0$, then $x = 2$.

In both cases, identify whether the statement is true or false. Justify your answer by giving a proof (if true), or a counterexample (if false). [5 marks]

9. A curve C has equation $y = -7x^{10} + 12x^5 + 4x^2$. Find the equation of the line tangent to C at $x = 1$. [5 marks]

10. Points A and B have position vectors $\mathbf{a} = 2\mathbf{i} - 3\mathbf{j}$ and $\mathbf{b} = \mathbf{i} + \mathbf{j}$.

(a) Find the distance between A and B . [3 marks]

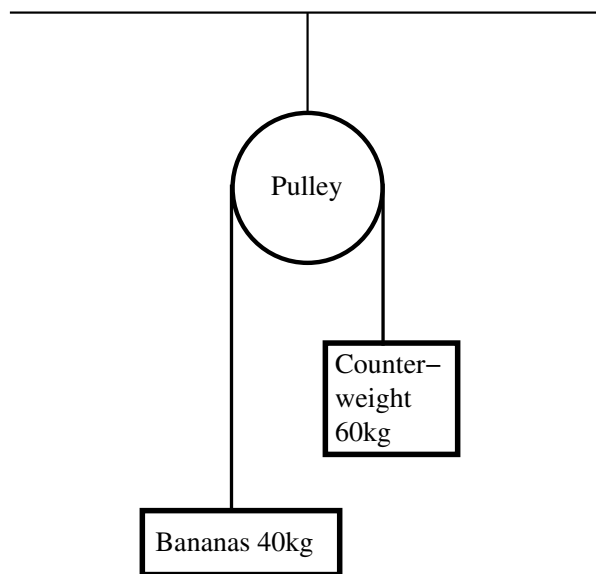
(b) Point C divides the line AB such that $AC : CB = 2 : 3$. Find, in terms of \mathbf{i} and \mathbf{j} , the position vector of C . [4 marks]

Section B

11. Consider the function

$$f(x) = -2x + 9x^2 - 9x^3.$$

- (a) Find all stationary points of the curve $y = f(x)$, stating your result using exact values simplified as far as possible. Make sure to indicate your method clearly, showing all working. [9 marks]
 - (b) Find the range of values of x for which $f'(x) \geq 0$, and hence (or otherwise) determine the nature of each stationary point found in (a). [7 marks]
 - (c) Find (i) the largest and (ii) the smallest value of the function $f(x)$ in the interval $0 \leq x \leq 1$, explaining your reasoning. [4 marks]
12. A 40kg crate of bananas is suspended from one end of an inextensible cable of negligible mass that passes over a frictionless pulley, and a 60kg counterweight is suspended from the other end; see figure below. The system is released from rest.



- (a) Sketch two free-body diagrams, one for the crate of bananas and one for the counterweight. In both cases, indicate all the forces acting on the object. (Numerical magnitudes are not required at this point.) [4 marks]
- (b) Find the magnitude of the upward acceleration of the crate. [8 marks]
- (c) Find (i) the magnitude of the tension force in the cable, and (ii) the ratio of the tension to the weight of the crate. [4 marks]
- (d) Find (i) the time it takes for the crate to reach the height of 2 metres from its initial position (assuming it does not reach the pulley by then), and (ii) the speed of the crate at that time. [4 marks]

13. (a) Suppose that in a game the possible scores are 1, 2, 3, 4, 5. The random variable, S , corresponding to a player's score on a particular game has the following probability distribution, where a and b are constants.

s	1	2	3	4	5
$P(S = s)$	a	0.1	0.3	b	0.2

Moreover, the probability that the score is 3 or less is equal to the probability of scoring at least 3 points, and each game played is independent of previous games played.

- (i) Find the values of a and b . [5 marks]
- (ii) Ahmed plays the game twice and adds the two scores together to get a total. Find the probability that his total is (strictly) greater than 7. [6 marks]
- (b) Researchers wish to estimate the proportion of people (in a population) who have a particular genetic condition. They sample 20 people (independently) and find that 9 of them have the condition.
- (i) If X is the number of people in a sample of size 20 that have the genetic condition and p is the true proportion of people with the condition, what is the distribution of X ? [2 marks]
- (ii) Carry out a hypothesis test to assess whether the true proportion of people with the condition is greater than 25%. Clearly show your working, including stating appropriate null and alternative hypotheses, and finding the P-value. State your conclusion using a 5% significance level. [7 marks]