

MATHEMATICS

Time allowed: 1 hour 30 minutes

- All answers (including any diagrams, graphs or sketches) should be written on paper, and scanned into a **single** PDF file. Graph paper is not required.
 - Answer **all** questions in Section A and **two** questions from Section B.
 - Candidates are permitted to use calculators, provided they comply with A level examining board regulations. They must be made available on request for inspection by invigilators, who are authorised to remove any suspect calculators.
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Additional information

- Statistical tables will be provided. Note that the tables refer to the **right-hand** tails of the distributions, that is, probabilities of the form $p = \mathbb{P}(X \geq x)$ where X is a random variable and x an **upper** percentage point of its distribution.
- Formulas related to standard distributions (e.g. for probability, mean, and variance) can be found on the back page of the statistical tables.

Section A

1. Simplify the following expressions as far as possible, showing your working clearly.

(a) $\left(\frac{1}{a+3} - \frac{2a+1}{a^2+a-6}\right)^{-1} + a;$ [6 marks]

(b) $1 + \frac{1}{1 + \frac{1}{1 + \sqrt{3}}}.$ [6 marks]

2. Under suitable conditions, bacterial growth may be modelled as follows: at time t hours, the number of bacteria in a sample is $N = Ce^{at}$ where C and a are constants.

(a) Explain the meaning of the constant C in this context. [1 mark]

(b) Suppose that the number of bacteria in the sample doubles every 4 hours. Find the value of a , to four decimal places. [4 marks]

(c) Find the time at which the sample contains 100 times the initial number of bacteria. [3 marks]

3. Solve the equation $\ln(1+x) = \ln x + 3$ for x (to 3 dp). [4 marks]

4. (a) Without using any decimal approximations, solve the equation

$$\left(\sin \theta - \frac{1}{\sqrt{2}}\right) \left(\sin \theta - \frac{\sqrt{3}}{2}\right) = 0.$$

for values of θ between 0° and 180° . [5 marks]

(b) Show that the equation in (a) can be written equivalently as

$$\sqrt{8} \cos^2 \theta + a \sin \theta = b,$$

and find the values of a and b . [4 marks]

5. Let $f(x) = 2x^3 + 3x^2 - 36x + 3$, for all real values of x .

(a) Find the range of values of x for which $f(x)$ is a decreasing function. [5 marks]

(b) Find the range of values of x for which the derivative $f'(x)$ is an increasing function. [2 marks]

6. In the following statements A and B, x and y are real numbers.

A If $(x-y)^2 = (2x-4y)^2$, then $x = 3y$.

B If $x = 3y$, then $(x-y)^2 = (2x-4y)^2$.

In both cases, identify whether the statement is true or false. Justify your answer by giving a proof (if true), or a counterexample (if false). [5 marks]

7. Find the indefinite integral $\int(2x^3 - x^{-5} + 3x^{-\frac{4}{5}} - x^{\frac{2}{3}})dx.$ [4 marks]

8. Points A and B have position vectors $\mathbf{a} = -\mathbf{i} + 3\mathbf{j}$ and $\mathbf{b} = 2\mathbf{i} - \mathbf{j}$.

- (a) Find the distance between A and B . [3 marks]
- (b) Point P divides the line AB such that $AP : PB = 3 : 5$. Find, in terms of \mathbf{i} and \mathbf{j} , the position vector of P . [4 marks]
- (c) Points C and D have position vectors $\mathbf{c} = \mathbf{b} + 6\mathbf{i} - 3\mathbf{j}$ and $\mathbf{d} = \mathbf{a} + 3\mathbf{i} + \mathbf{j}$. Find the vector \mathbf{CD} and determine whether it is parallel to \mathbf{AB} , explaining your reason. [4 marks]

Section B

9. Curve C has equation

$$x^2 + y^2 - 8x + 6y = 0.$$

- (a) Show that C is a circle by rewriting the equation in a suitable equivalent form, and state the radius and the coordinates of the centre of C . [6 marks]
- (b) Show that C passes through the origin. [1 mark]
- (c) Point P lies on the circle C diametrically opposite to the origin. State the coordinates of P . Justify your answer by sketching a figure showing the relevant geometry. [2 marks]
- (d) Find the points at which C intersects the line $y = 3x - 20$. [6 marks]
- (e) Show that the point $(1, 1)$ lies on the circle C , and find the equation of the line tangent to C at this point. [5 marks]

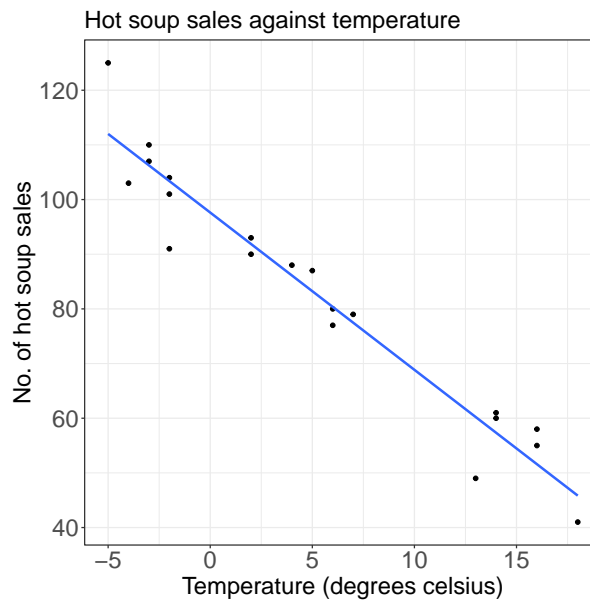
10. Three blocks are being pulled horizontally on a frictionless surface of ice using horizontal ropes (of negligible mass) aligned with the direction of the pull and remaining taut, as shown in the diagram below. The resulting acceleration of the three blocks is 3 m/s^2 in magnitude.



- (a) (i) Considering the three blocks as a composite object, sketch a free-body diagram for it, that is, indicate all the forces acting on the object. (Numerical magnitudes are not required at this point.) [2 marks]
- (ii) Compute the magnitude of the pulling force, stating clearly the relevant equation of motion based on your free-body diagram in (i). [2 marks]
- (b) (i) Sketch a separate free-body diagram for each individual block, showing all the forces acting on the block. (Numerical magnitudes are not required at this point.) [7 marks]
- (ii) Hence compute the tension forces in the two ropes between the blocks, starting from equations of motion based on **two** of the three diagrams in (i). Indicate clearly which diagrams you use. [6 marks]
- (iii) Verify that the equation of motion based on the **remaining** diagram is now satisfied by the tension forces you obtained in (ii). [3 marks]

Note: to get full marks in part (b) (ii)-(iii) your working must be sufficient to show consistency with the free-body diagrams your reasoning is based on.

11. (a) Researchers are interested in establishing whether the proportion of men in the population who are colour-blind is larger than 10%. In a random sample of 50 men, 9 were found to be colour-blind. Stating clearly your null hypothesis and alternative hypothesis, and the distribution of the relevant test statistic, assess the evidence (at the 5% level) that the proportion who are colour-blind is greater than 10%. [9 marks]
- (b) A catering van business was interested in the relationship between the outside air temperature and the number of sales of hot soup portions. They collected data on the temperature and the number of hot soup sales on each of 20 days spread out through the year. They obtained the following plot.



The equation of the fitted line is

$$\text{soup sales} = 97.6 - 2.9 \times \text{temperature}.$$

- (i) Describe the relationship between temperature and number of hot soup sales, including the use of the word *correlation*. [3 marks]
- (ii) What is the predicted number of hot soup sales at a temperature of 10 degrees celcius? [2 marks]
- (iii) Explain clearly what the two numbers in the equation of the fitted line mean. [4 marks]
- (iv) Suppose that for a second product, changes in temperature have less (but still a decreasing) effect on sales, and the amount sold at 0 degrees celsius is 120 units. Describe how the fitted line would look like for this second product (in comparison to the first) for temperatures greater than or equal to 0 degrees celsius. [2 marks]