Interval type-2 fuzzy decision making

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Based on..

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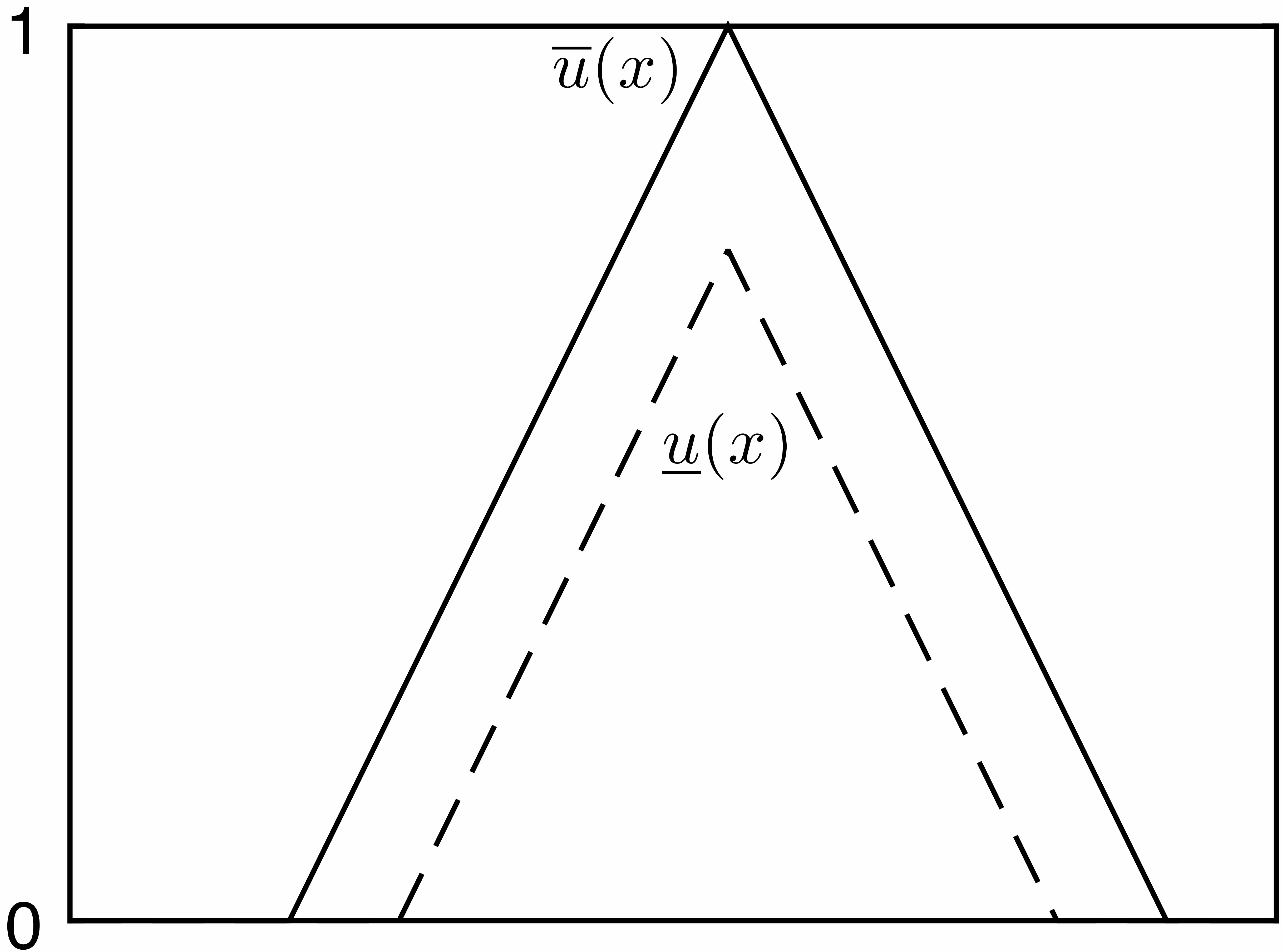
People

Thomas Runkler is an extremely well known fuzzy logic researcher who works at Siemens in Germany.

Simon Coupland is based at De Montfort University and has also published widely on fuzzy logic

What is this talk about?

Decision making under uncertainty and the consideration of risk.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Motivation

A. Previous work considered the role of type-2 defuzzification in decision making[1].

B. Particularly the semantic meaning of defuzzified values from the perspective of opportunity or risk.

C. Interested in the notion of risk and how different individuals make decisions.

1 - T. A. Runkler, S. Coupland and R. John, "Properties of interval type-2 defuzzification operators," Fuzzy Systems (FUZZ-IEEE), 2015 IEEE International Conference on, Istanbul, 2015, pp. 1-7.

What do we mean by decision making?

- We have **goals** that are limited by **constraints.**
- In the case of fuzzy decision making under uncertainty we wish to find an optimal decision when goals and constraints are represented by **fuzzy sets.**

Bellman, R., Zadeh, L., 1970. Decision making in a fuzzy environment. Management Science 17 (4), 141–164.

Some definitions

A type–1 fuzzy set A is defined by a membership function

$$u_A : X \to [0, 1].$$

Consider fuzzy sets over one–dimensional continuous intervals

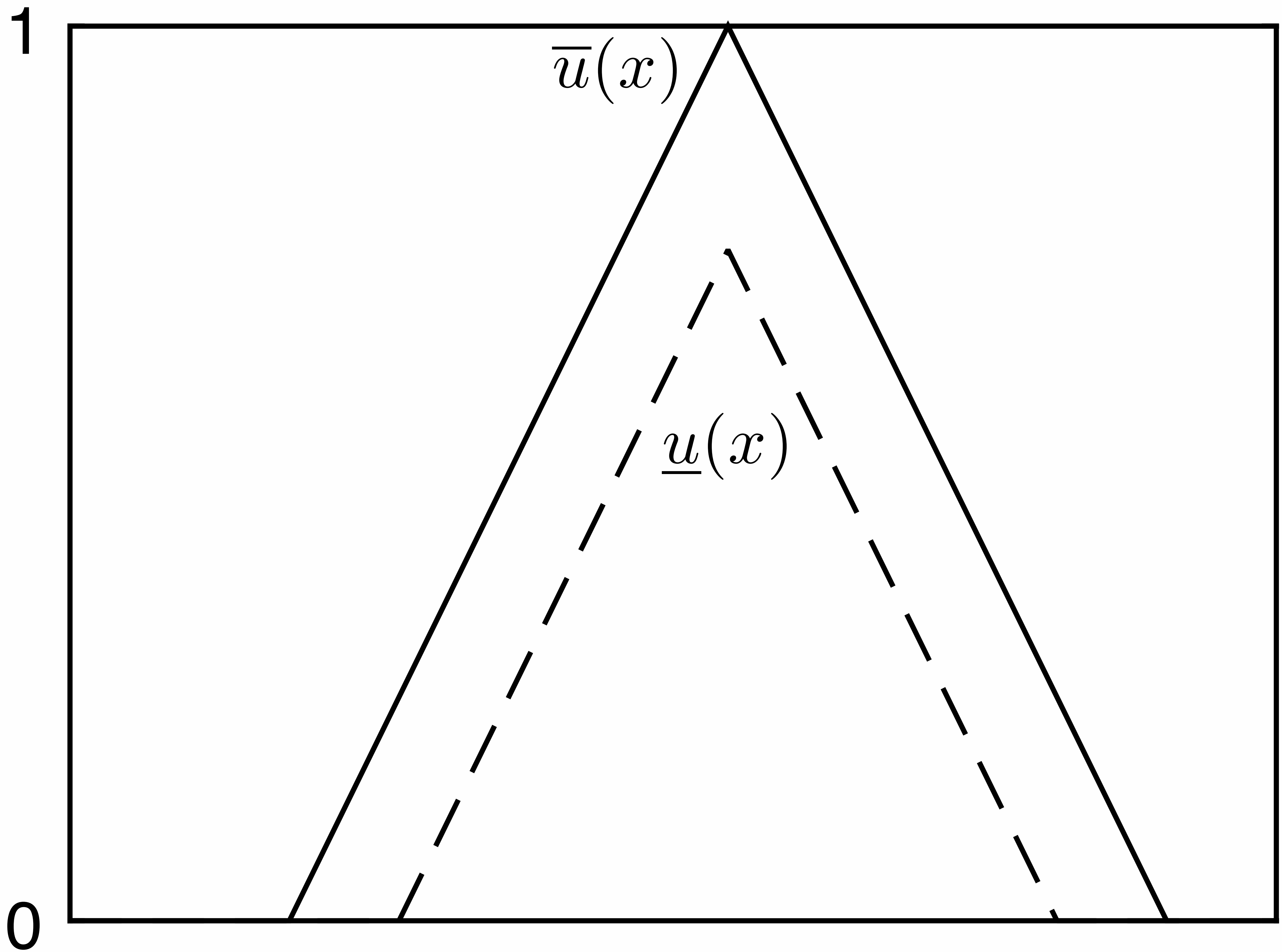
$$X = [x_{\min}, x_{\max}].$$

Interval type-2 fuzzy set

An interval type–2 fuzzy set $\tilde{A}$ is defined by two membership functions:
a lower membership function $\underline{u}_{\tilde{A}} : X \to [0, 1]$
and an upper membership function $\overline{u}_{\tilde{A}} : X \to [0, 1]$, where

$$\underline{u}_{\tilde{A}}(x) \leq \overline{u}_{\tilde{A}}(x)$$

for all $x \in X$.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Type-1 Fuzzy Decision Making

Given a set of goals specified by the membership functions

$$\{u_{g_1}(x), \ldots, u_{g_m}(x)\}$$

and a set of constraints specified by the membership functions

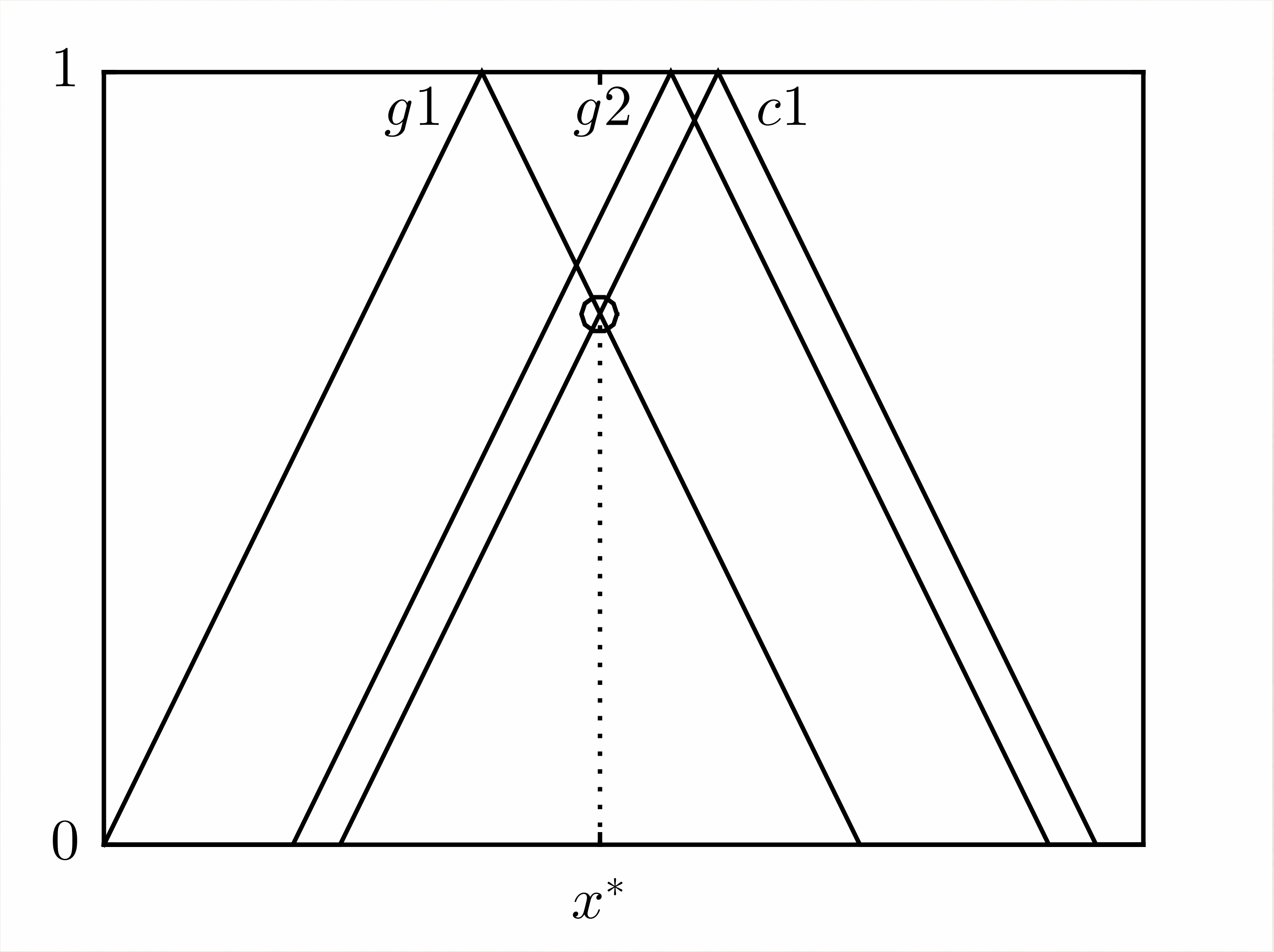
$$\{u_{c_1}(x), \ldots, u_{c_n}(x)\}$$

the optimal decision x^* is defined as

$$x^* = \underset{x \in X}{\operatorname{argmax}} \left(u_{g_1}(x) \wedge \ldots \wedge u_{g_m}(x) \; \wedge \; u_{c_1}(x) \wedge \ldots \wedge u_{c_n}(x) \right)$$

where $\wedge$ is a triangular norm such as the minimum or the product operator.

Type-1 fuzzy decision making



Risk and decision making

- Previous approaches to type-2 decision making
  - Multi criteria decision making
  - Ranking
- Don't take account of attitude to risk
- **We are interested in combining risk and decision making in an interval type-2 framework**

Type-1 fuzzy decision making

- Membership values of goals and constraints represent utility of decision making
- But these are ‘crisp’ and do not reflect any uncertainty in the utility
- In interval type-2 fuzzy decision making the utilities are assumed to be uncertain (intervals)
- The upper bound represents the ‘best’ case and the lower bound the ‘worst’ case of each utility

Worst case decision

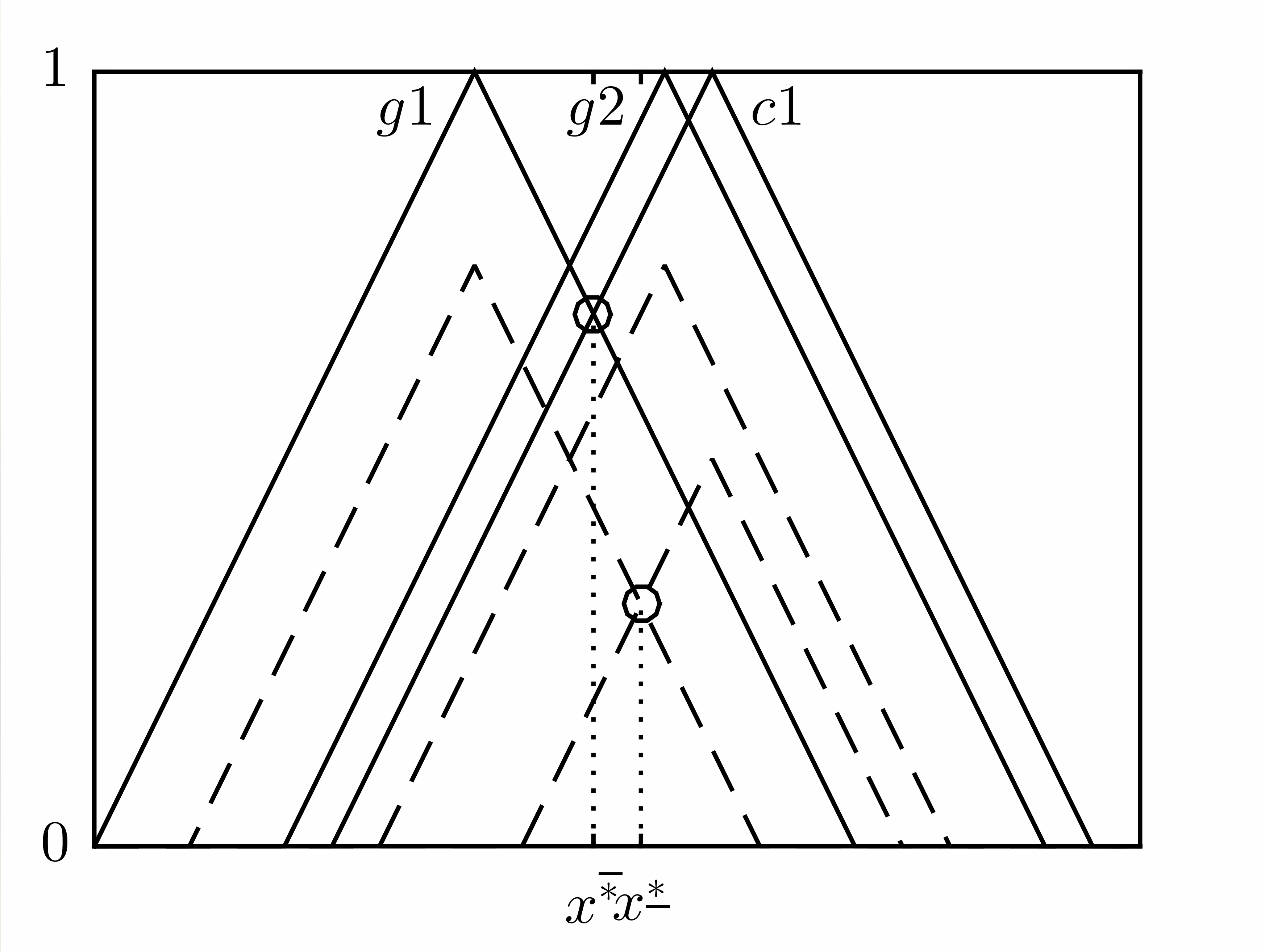
$$x^{\underline{*}} = \underset{x \in X}{\operatorname{argmax}} \left(\underline{u}_{\tilde{g_1}}(x) \wedge \ldots \wedge \underline{u}_{\tilde{g_m}}(x) \ \wedge \ \underline{u}_{\tilde{c_1}}(x) \wedge \ldots \wedge \underline{u}_{\tilde{c_n}}(x) \right)$$

Best case decision

$$x^{\overline{*}} = \underset{x \in X}{\operatorname{argmax}} \left(\overline{u}_{\tilde{g_1}}(x) \wedge \ldots \wedge \overline{u}_{\tilde{g_m}}(x) \ \wedge \ \overline{u}_{\tilde{c_1}}(x) \wedge \ldots \wedge \overline{u}_{\tilde{c_n}}(x) \right)$$

What does this mean?

- The worst case interval type-2 fuzzy decision maximises the utility that is obtained under the worst possible conditions.
- This decision policy reflects a cautious or pessimistic decision maker.
- The best case interval type-2 fuzzy decision maximises the utility that is obtained under the best possible conditions.
- This decision policy reflects a risky or optimistic decision maker.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0
g1
g2
c1
x*x*

- We do not want to restrict the interval type--2 fuzzy decision to the worst case and best case decisions
- We want to allow to specify the level of risk $\beta \in [0, 1]$
- Where risk $\beta = 0$ corresponds to the worse case decision and $\beta = 1$ corresponds to the best case decision

The interval type-2 fuzzy decision at risk level β

$$x^*_\beta = \operatorname*{argmax}_{x \in X} \Big(((1-\beta)\cdot \underline{u}_{\tilde{g_1}}(x) + \beta \cdot \overline{u}_{\tilde{g_1}}(x))$$

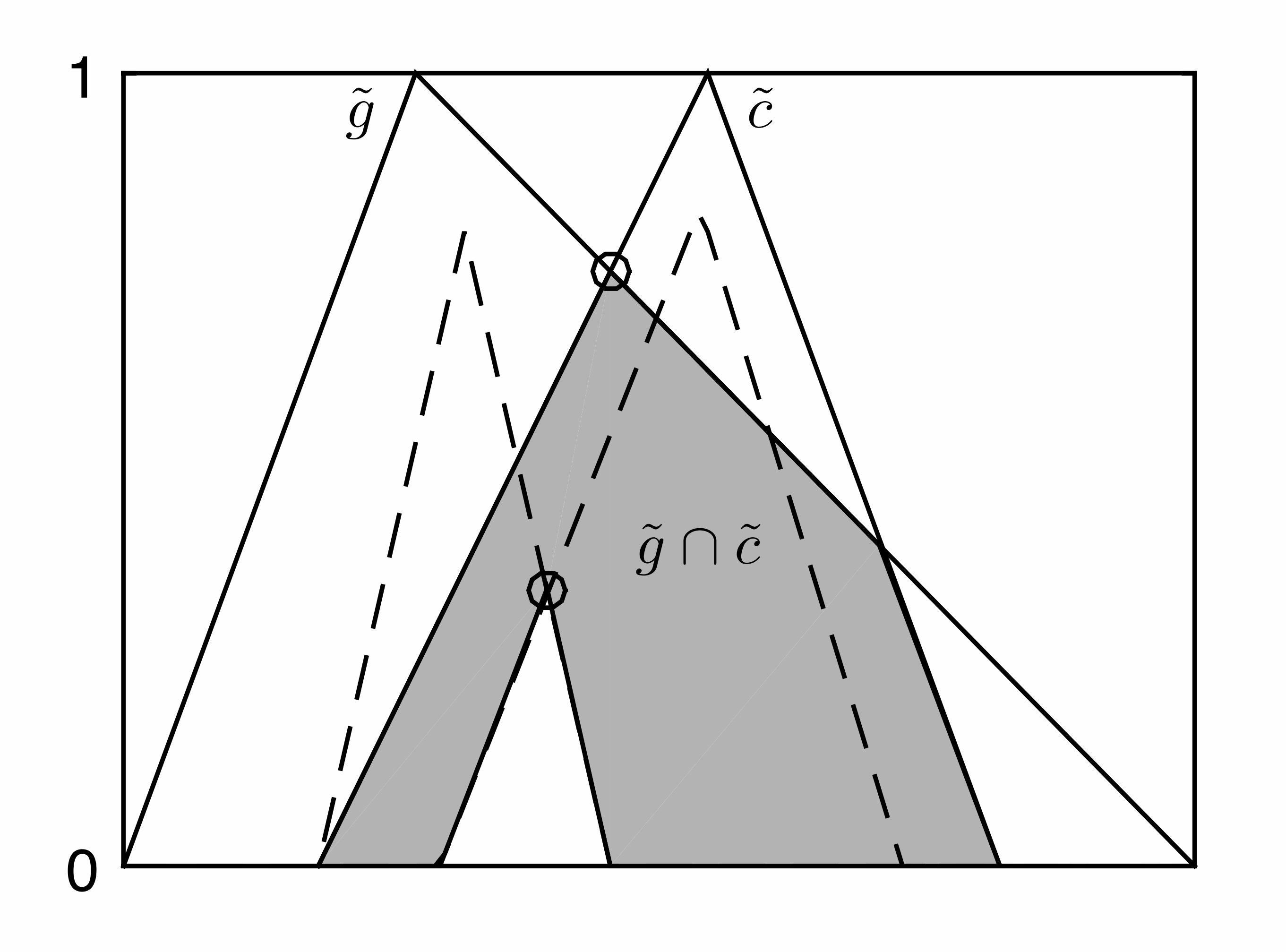
$$\wedge \ldots \wedge ((1-\beta)\cdot \underline{u}_{\tilde{g_m}}(x) + \beta \cdot \overline{u}_{\tilde{g_m}}(x))$$

$$\wedge ((1-\beta)\cdot \underline{u}_{\tilde{c_1}}(x) + \beta \cdot \overline{u}_{\tilde{c_1}}(x))$$

An alternative view

- The worst case formula is to find the maximum value across the domain from the minimum of all the membership functions at a domain point x.
- This could equally be obtained by finding the highest membership grade across the domain of a fuzzy set which is the intersection of all goals and constraints.

$$\tilde{f} = \tilde{g_1} \cap \ldots \cap \tilde{g_m} \cap \tilde{c_1} \cap \ldots \cap \tilde{c_n}$$



The approach leads to these equations giving alternative ways of calculating the respective worst and best case decisions.

$$x^{*}_{-} = \operatorname*{argmax}_{x \in X}(\underline{\tilde{f}}(x))$$

$$x^{\overline{*}} = \operatorname*{argmax}_{x \in X}(\overline{\tilde{f}}(x))$$

$$x^{*}_{\beta} = \operatorname{argmax}\left((1-\beta)\cdot\mu_{\underline{\tilde{f}}}(x) + \beta\cdot\mu_{\overline{\tilde{f}}}(x))\right)$$

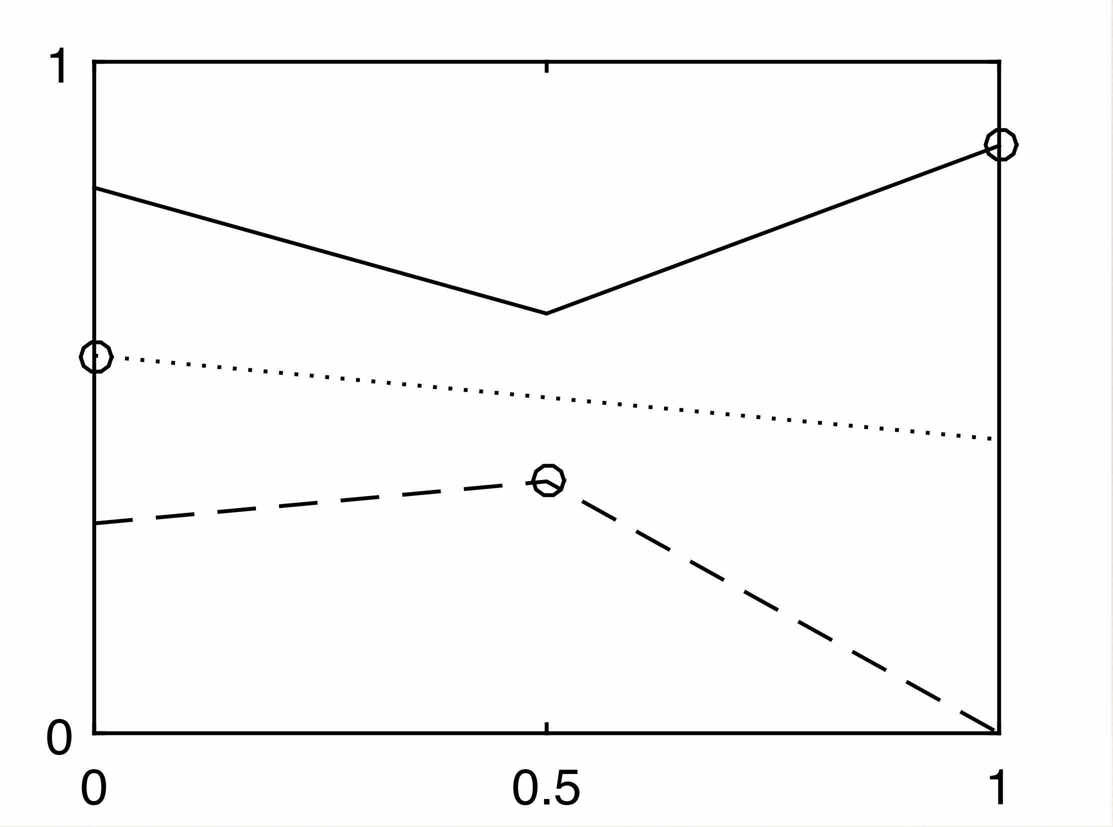
Properties of Interval Type-2 Fuzzy Decision Making

It seems reasonable to require that for any risk level $\beta \in [0,1]$ the decision should be in the interval bounded by the worst case decision x^*_{-} and the best case decision $x^{\overline{*}}$, so

$$\min\left(x^*_{-}, x^{\overline{*}}\right) \le x^*_\beta \le \max\left(x^*_{-}, x^{\overline{*}}\right)$$

for arbitrary t–norms $\wedge$.

- We now check whether the previous equation holds for all fuzzy sets (it's always a contentious issue in the fuzzy world!)



If the membership functions are convex?

We know that taking the minimum or the product of two convex functions will always yield a convex function.

Consider the two convex interval type-2 fuzzy sets $\tilde{g_1}$ and $\tilde{c_1}$ over the domain X.

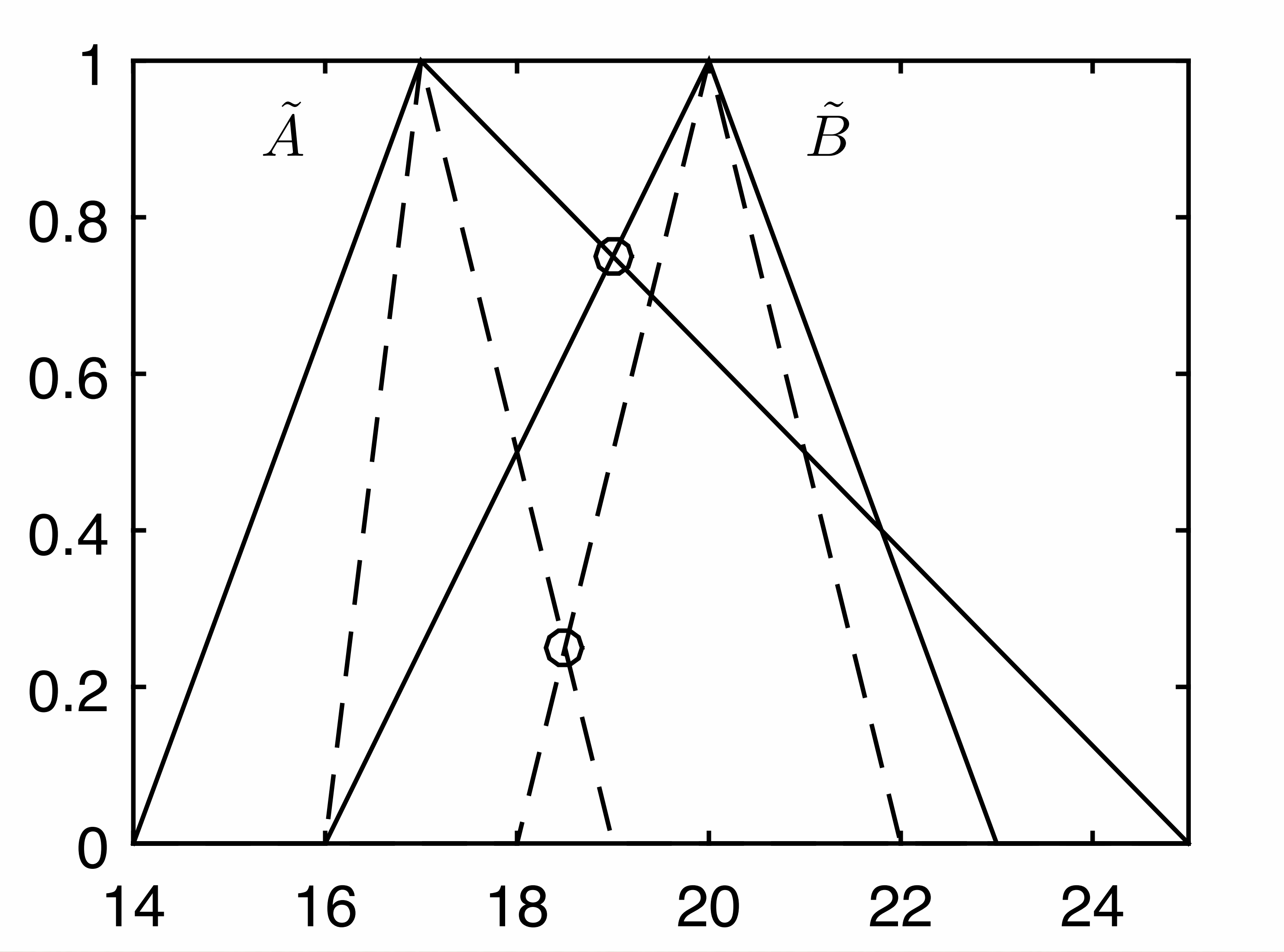
$\underline{\tilde{g_1}} \cap \underline{\tilde{c_1}}$ and $\overline{\tilde{g_1}} \cap \overline{\tilde{c_1}}$ must yield convex functions when using the product or minimum t-norm. Let $\tilde{f} = \tilde{g_1} \cap \tilde{c_1}$

It is obvious that the lower membership function of $\tilde{f}$ is contained by the upper membership function of $\tilde{f}$ i.e. $\overline{\tilde{f}}(x) \geq \underline{\tilde{f}}(x), \forall x \in X$.

Application examples

- 2 guests staying with you (A and B)
- Problem: How to set the room temperature?
- A will be completely happy with 17 degrees, and will be completely unhappy at less than 16 degrees or more than 19 degrees.
- B will be completely happy with 20 degrees, and will be completely unhappy for less than 18 degrees or more than 22 degrees.

Here's what the interval set could look like

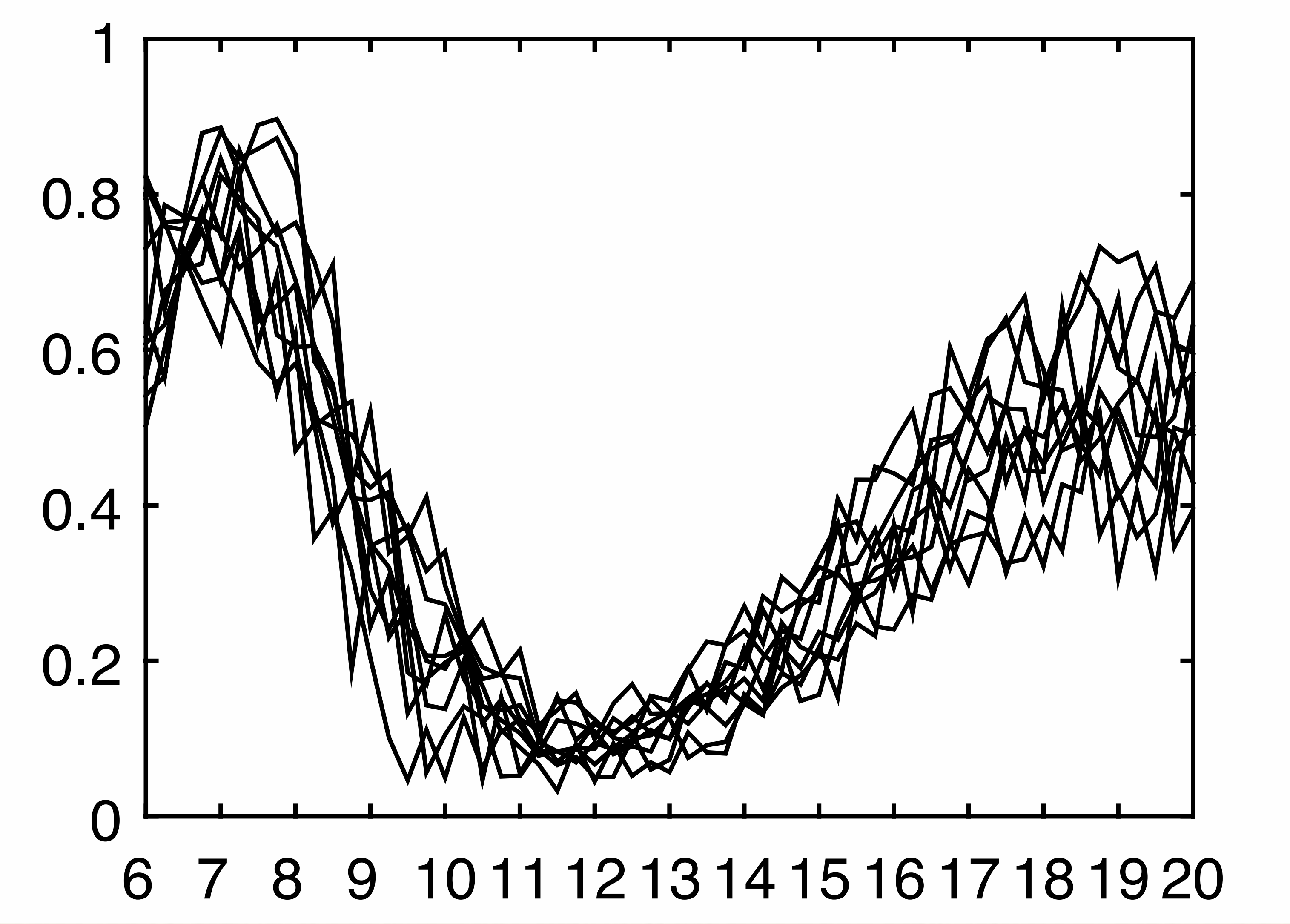


So….

- Cautious decision maker will set the temperature to 18.5 degrees because then none of the guests will be less happy than 25%.
- Risky decision maker will 19 degrees because in the best case both guests will be 75% happy.
- Intermediate levels of risk will yield optimal temperatures between 18.5 and 19 degrees.

Driving to work

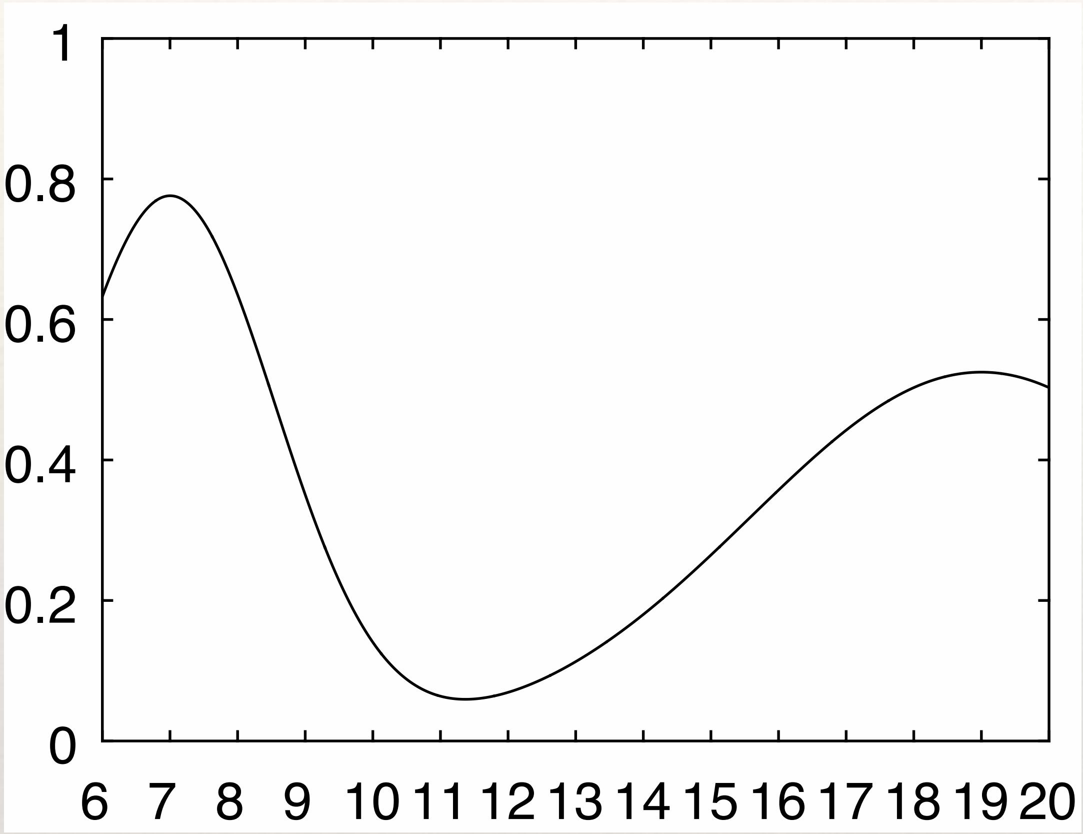
- We want to drive to work at some time between 6 and 12 o'clock, work for 8 hours, and then drive back.

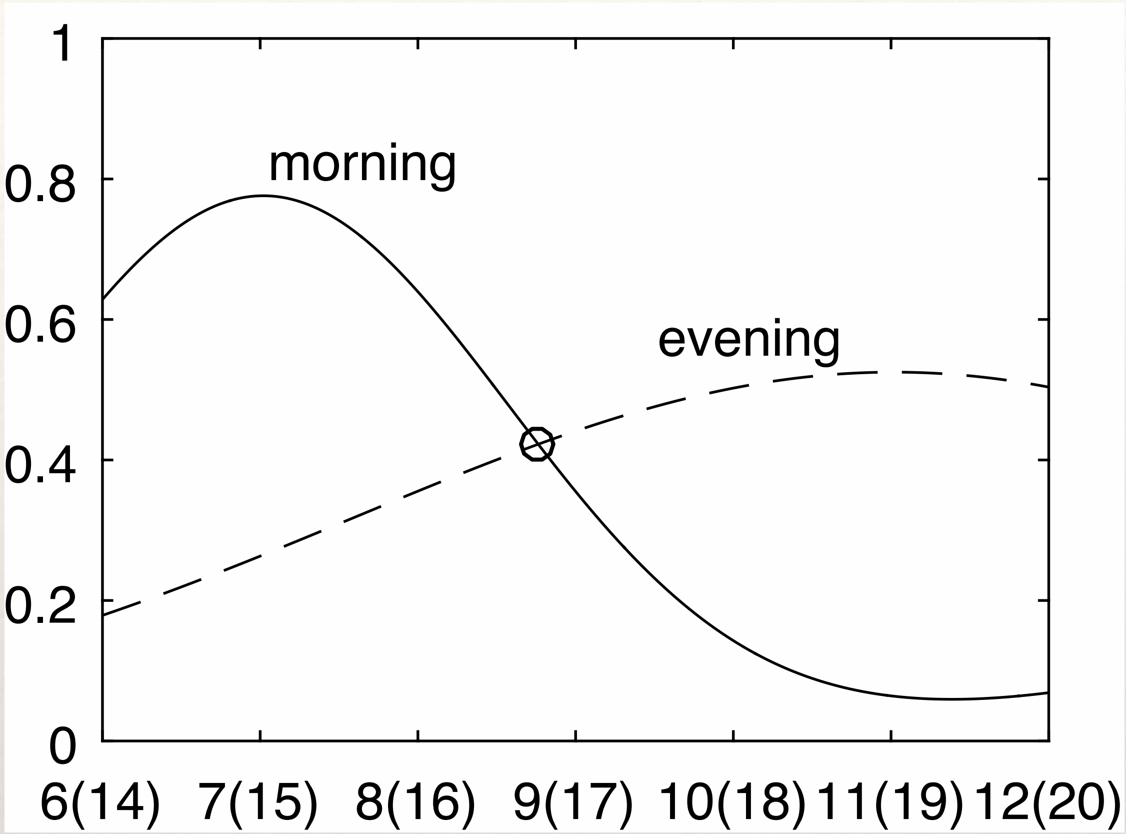


- Based on the observed traffic densities we estimate the average traffic densities using a mixture of two Gaussian membership functions as:

$$u(x) = 0.775 \cdot e^{-\left(\frac{x-7\cdot 60}{133}\right)^2} + 0.525 \cdot e^{-\left(\frac{x-19\cdot 60}{290}\right)^2}$$

Type-1 approach





Type-1 fuzzy set "Traffic"

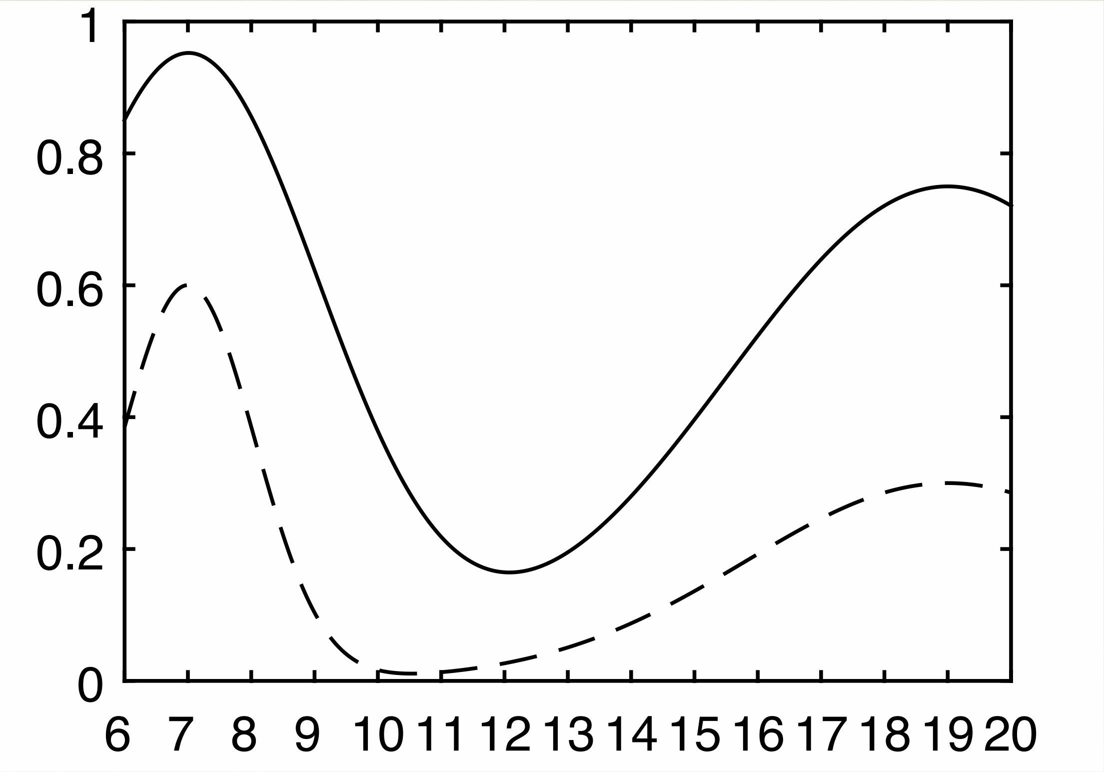
If we do the morning trip at 7:00 and the evening trip at 15:00, for example, then we will have 0.775 traffic in the morning and about 0.26 traffic in the evening. Our goal is to find a travel time, where the traffic in the morning is low and the traffic in the evening is low.

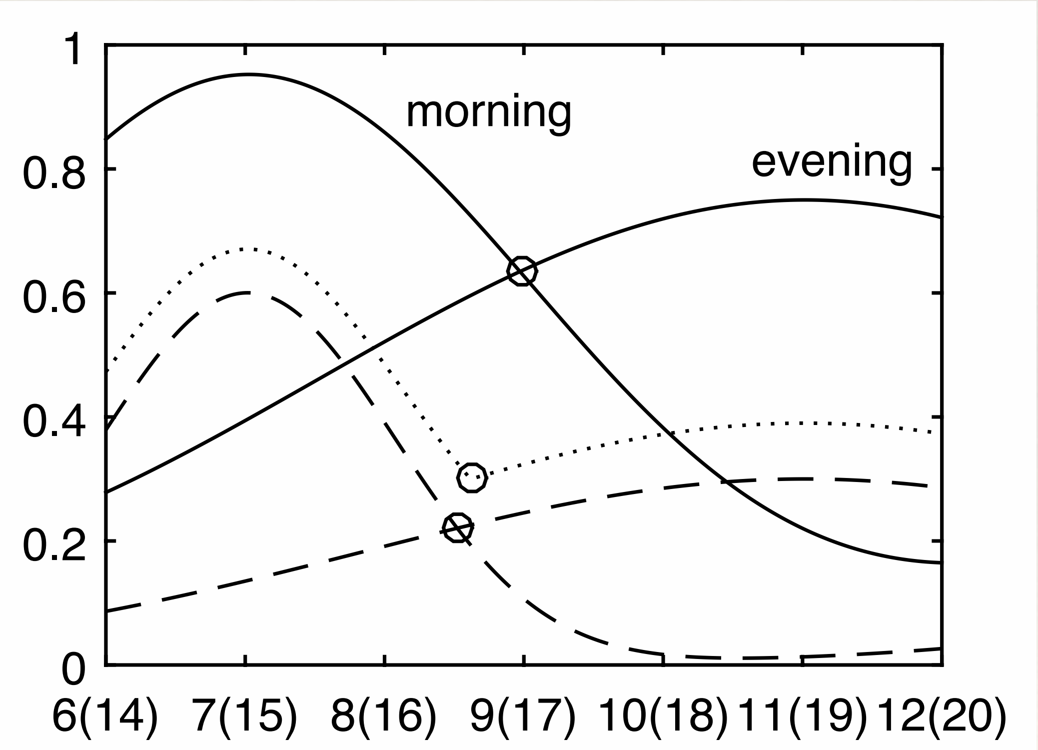
- This time we need to minimise so we use argmin
- The optimal type--1 fuzzy decision (marked by a circle) is at 8:46 (return 16:46) with a traffic of 0.42 for both the morning and the evening trips.

A type-2 version

$$\overline{u}(x) = 0.95 \cdot e^{-\left(\frac{x-7\cdot 60}{3\cdot 60}\right)^2} + 0.75 \cdot e^{-\left(\frac{x-19\cdot 60}{3\cdot 60}\right)^2}$$

$$\underline{u}(x) = 0.6 \cdot e^{-\left(\frac{x-7\cdot 60}{1.5\cdot 60}\right)^2} + 0.3 \cdot e^{-\left(\frac{x-19\cdot 60}{4.5\cdot 60}\right)^2}$$

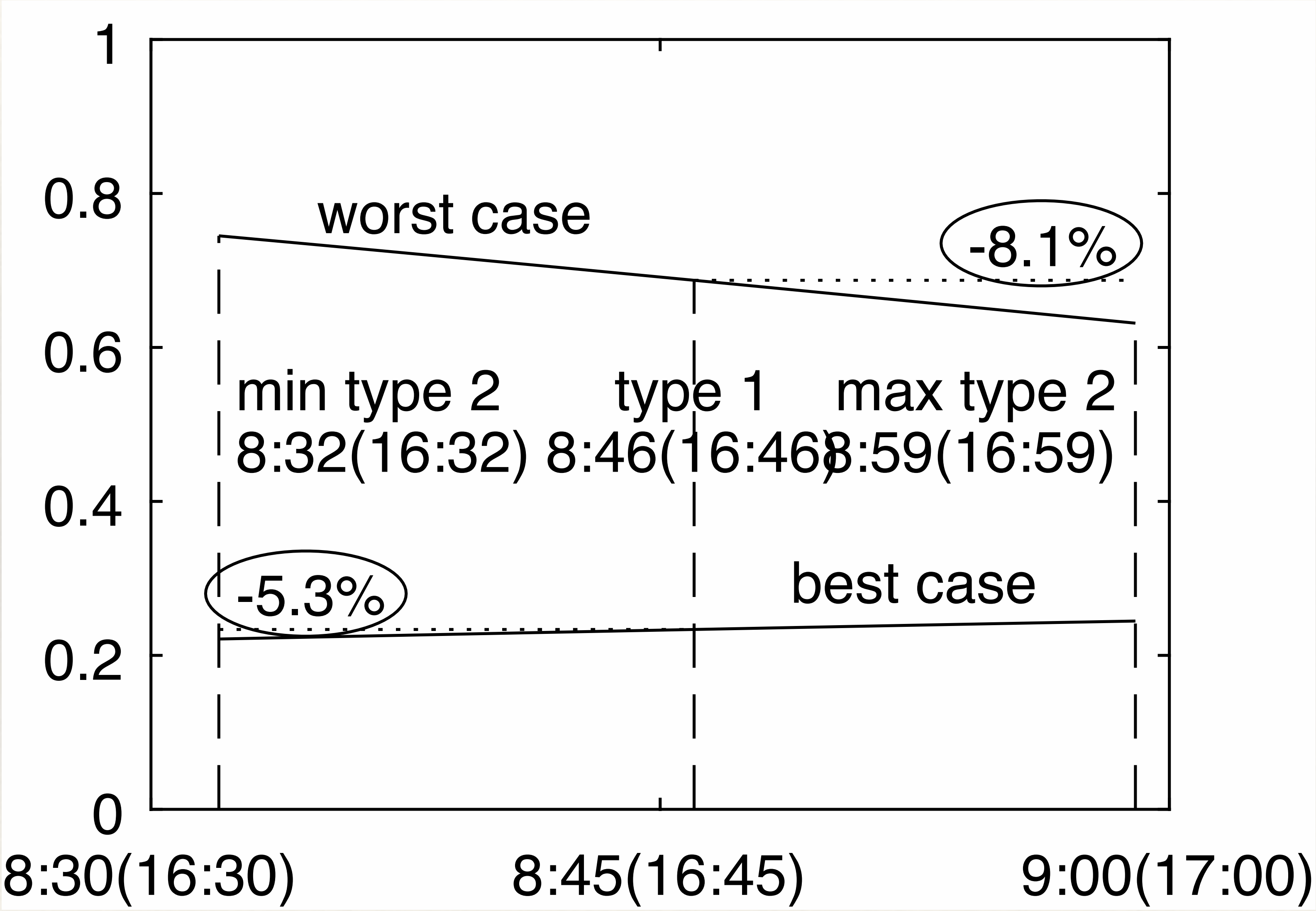




Decisions.....

- A cautious decision maker will drive to work at 8:59 and back at 16:59 (upper circle), because the worst case traffic is about 0.64.
- A risky decision maker will drive to work at 8:32 and back at 16:32 (lower circle), because the best case traffic is about 0.22.
- For intermediate levels of risk the optimal decision will be to leave between 8:32 and 8:59 and return 8 hours later.
- For example, for risk level β =0.8 we obtain the dotted curve which is minimised for leaving at 8:37 and returning at 16:37 with a traffic of about 0.3.

A comparison



Conclusions

- Existing approaches supporting decision making using type-2 fuzzy sets ignore the risk associated with these decisions.
- We have presented a new approach to using interval type--2 fuzzy sets in decision making with the notion of risk. This brings an extra capability to model more complex decision making, for example, allowing trade-offs between different preferences and different attitudes to risk.
- The explicit consideration of risk levels increases the solution space of the decision process and thus enables better decisions.
- In a traffic application example, the quality of the obtained decision could be improved by 5.3-8.1%.

References

- T. A. Runkler, S. Coupland and R. John, "Properties of interval type-2 defuzzification operators," Fuzzy Systems (FUZZ-IEEE), 2015 IEEE International Conference on, Istanbul, 2015, pp. 1-7.
- Thomas Runkler, Simon Coupland, Robert John, Interval type-2 fuzzy decision making, International Journal of Approximate Reasoning, Volume 80, January 2017, Pages 217-224, ISSN 0888-613X.

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